MATH 311H: Homework 4

Due: October 2 at 5 pm

1. Upcoming office hours are Monday September 25 and Thursday September 28 10-11 am in LSH B-102D.

2. Read Sections 2.3-5 in Abbott.

3. Do Abbott Exercise 1.5.1, 1.5.9*, 2.2.2*, 2.3.2, 2.3.4, 2.3.8

4. Recall that in class we said that two sets A and B have the same size, or cardinality, if there is a bijective function $f : A \to B$, and wrote $A \sim B$. Confirm that (as the notation implies) this is an equivalence relation.

5. Find bijections between the following pairs of sets.

   (a) $[0, 1)$ and $[0, 1]$.*
   (b) The interval (a, b) for $a < b$ and $\mathbb{R}$.
   (c) The set S consisting of sequences $(s_i)_{i=0}^{\infty}$ such that s_i is 0 or 1 and the set of subsets of the natural numbers.

6. Consider the construction of the real line as Dedekind cuts given in class. Prove that if r and s are two positive rationals and r^* and s^* are their inclusions into $\mathbb{R}$, then $r^*s^* = (rs)^*$. That is, show that the definition of multiplication of Dedekind cuts that we gave in class agrees with our pre-existing definition of multiplication on the rationals.*

   [The proof for addition is done in the notes; it may help to read it before working on multiplication.]