

# Math 311: Quiz

February 8, 2021

## Instructions

You have thirty minutes to take the quiz. There are three questions, each of which is worth five points. You should not use any notes, books, websites, or other aids. After time is called, please upload your solutions, after which you will be asked to record a brief video of yourself explaining one of your solutions for authentication purposes.

### Problem 1

Use induction to prove that  $7^n - 6n - 1$  is divisible by 36 for all  $n \geq 2$ .

### Problem 2

If  $A$  and  $B$  are sets, their *product* is the set  $A \times B = \{(a, b) : a \in A, b \in B\}$ . Prove that if

$$\begin{aligned} A &= \{a_1, a_2, a_3, \dots\} \\ B &= \{b_1, b_2, b_3, \dots\} \end{aligned}$$

are both countable sets, then their product  $A \times B = \{(a_i, b_j) : i, j \in \mathbb{N}\}$  is as well. (Hint: Your proof should look similar to the proof that  $\mathbb{Q}$  is countable from class.)

### Problem 3

Let  $A$  be a nonempty subset of  $\mathbb{R}$  which is bounded above. Let  $s$  be a real number with the property that if  $n \in \mathbb{N}$ , then  $s + \frac{1}{n}$  is an upper bound for  $A$  and  $s - \frac{1}{n}$  is not an upper bound for  $A$ . Prove that  $s = \sup A$ .