Math 311: Sample Midterm 1

March 31, 2021

Instructions

You have sixty minutes to take the exam. There are five questions, each of which is worth five points. You should not use any notes, books, websites, or other aids. After time is called, please upload your solutions, after which you will be asked to record a brief video of yourself explaining one of your solutions for authentication purposes.

Problem 1

For each of the following, give an example of the object described (no need to justify your work) or say why this is impossible.

(a) A perfect set consisting only of rational numbers.

(b) A sequence of open sets $F_1 \supseteq F_2 \supseteq F_3 \supseteq \dots$ such that $\bigcap_{n=1}^{\infty} F_n$ is empty.

(c) A closed set E for which $\overline{(E^\circ)} \neq E$.

(d) A bounded above set A which contains its supremum but is not closed.

Problem 2 Let A_n be a subset of $\mathbb{R}$ for every n.

(a) Prove that $\bigcup_{n=1}^{\infty} A_n^\circ \subseteq (\bigcup_{n=1}^{\infty} A_n)^\circ$.

(b) Give an example to show that the inclusion above need not be an equality of sets.

Problem 3

Let $g : \mathbb{R} \to \mathbb{R}$ be a continuous function. Prove that if $g(r) = 0$ for all points $r \in \mathbb{Q}$, then in fact $g(x) = 0$ for all $x \in \mathbb{R}$.

Problem 4

Suppose that f, g are two functions with the same domain A such that $f(x) \leq g(x)$ for all $x \in A$, and say that $\lim_{x\to c} f(x) = L_1$ and $\lim_{x\to c} L_2$ both exist for some limit point c of A. Prove that $L_1 \leq L_2$.

Problem 5

For each of the following pairs of sets, either give an example of a continuous function $f : A \to \mathbb{R}$ whose image is $f(A) = B$ (no need to justify your answer) or explain why no such function exists.

(a) $A = [0, 1]$; $B = [1, 2)$

(b) $A = (0, 1]$; $B = [1, \infty)$

(c) $A = (0, 1)$; $B = (0, 1) \cup (3, 4)$