Math 311: Sample Midterm 1

February 18, 2021

Instructions

You have sixty minutes to take the exam. There are five questions, each of which is worth five points. You should not use any notes, books, websites, or other aids. After time is called, please upload your solutions, after which you will be asked to record a brief video of yourself explaining one of your solutions for authentication purposes.

Problem 1

For each of the following things, either give an example of the described object (no need to justify it) or write a sentence saying why this is impossible.

(a) [1 pt] A sequence which is not monotone and has no convergent subsequence.

(b) [1 pt] A Cauchy sequence with a divergent subsequence.

(c) [1 pt] An alternating series $\sum_{n=1}^{\infty} a_n$ of rational numbers converging to $\sqrt{2}$, and a partial sum of this series which is within .01 of $\sqrt{2}$.

(d) [1 pt] A sequence with exactly two subsequential limits.

(e) [1 pt] A bounded sequence with no Cauchy subsequence.

Problem 2

Let (a_n) be a bounded sequence and (b_n) be a sequence such that $b_n \to 0$. Prove that $a_n b_n \to 0$. [Warning: The Algebraic Limit Theorem doesn't apply to this situation.]

Problem 3

Suppose that a_n and b_n are Cauchy sequences. Prove directly that $a_n + b_n$ is a Cauchy sequence. ["Directly" means your proof should not reference the fact that Cauchy sequences converge in $\mathbb{R}$.]

Problem 4 Consider the sequence defined recursively by $a_0 = 1$ and $a_{n+1} = 2(a_n)^{\frac{2}{3}}$. Prove that this sequence converges and find the limit.

Problem 5

Let $\sum_{n=1}^{\infty} a_n$ be a series with the property that $\lim |a_n|^{\frac{1}{n}}$ exists and is equal to $L < 1$. Prove that $\sum_{n=1}^{\infty} a_n$ converges absolutely. [This result is called the *Root Test*.]