MATH 311: Homework 4

Due: February 17, 2021

1. Upcoming office hours are Monday February 15 3:30-4:30 and Wednesday February 17 9:00-10:00

2. Read Sections 2.5-6 in Abbott.

3. Do Abbott Exercise 2.3.3, 2.3.5, 2.3.10

4. Assume that \((a_n)\) is a sequence with \(a_n \neq 0\) for which the limit \(L = \lim \frac{a_{n+1}}{a_n}\) exists.
   
   (a) Show that if \(L < 1\) then \(\lim a_n = 0\). Hint: Choose \(k\) such that \(L < k < 1\). Then there exists \(N\) such that \(n \geq N\) implies that \(\frac{a_{n+1}}{a_n} < k\), so \(|a_{n+1}| < k|a_n|\) for all \(n \geq N\).

   (b) Show that \(\lim_{n \to \infty} \frac{a^n}{n^p} = 0\) for all \(|a| < 1\) and \(p > 0\).

   (c) Show that \(\lim_{n \to \infty} \frac{a^n}{n!} = 0\) for all \(a \in \mathbb{R}\).

5. Do Abbott Exercises 2.4.1, 2.4.2, 2.4.8