MATH 311: Homework 4

Due: February 17, 2021

1. Upcoming office hours are Monday February 15 3:30-4:30 and Wednesday February 17 9:00-10:00
2. Read Sections 2.5-6 in Abbott.
3. Do Abbott Exercise 2.3.3, 2.3.5, 2.3.10
4. Assume that (a_n) is a sequence with $a_n \neq 0$ for which the limit $L = \lim \left| \frac{a_{n+1}}{a_n} \right|$ exists.
   (a) Show that if $L < 1$ then $\lim a_n = 0$. Hint: Choose k such that $L < k < 1$. Then there exists N such that $n \geq N$ implies that $\left| \frac{a_{n+1}}{a_n} \right| < k$, so $|a_{n+1}| < k|a_n|$ for all $n \geq N$.
   (b) Show that $\lim_{n\to\infty} \frac{a^n}{n^p} = 0$ for all $|a| < 1$ and $p > 0$.
   (c) Show that $\lim_{n\to\infty} \frac{a^n}{n!} = 0$ for all $a \in \mathbb{R}$.
5. Do Abbott Exercises 2.4.1, 2.4.2, 2.4.8