MATH 311: Homework 1

Due: January 27, 2021

1. Send me an email introducing yourself! Tell me your favorite thing you've learned in a math class previously, and anything about yourself you think it would be useful for me to know. (Alternately, come by office hours and tell me these things.)

2. Upcoming office hours are Wednesday January 20 3:30-4:30, Monday January 25 3:30-4:30, and Wednesday January 27 9:00-10:00.

3. Read Sections 1.1-1.4 in Abbot.

4. Do Abbot Exercises 1.2.1, 1.2.2, and 1.2.12.

5. Prove that $(11)^n - 4^n$ is divisible by 7 for all positive integers n.

6. Prove Bernoulli's inequality: For $x \in \mathbb{R}$ with $x > 0$, and every natural number $n > 1$, $(1+x)^n > 1 + nx$.

7. Incorrect Inductions

   (a) Consider the following inductive "proof" that all horses are the same color. We will show that any set of n horses have the same color. The base case is trivial, since any set consisting of a single horse has only one color. Now suppose that all sets of $n-1$ horses have only one color. Then if $A = \{x_1, \cdots x_n\}$ is a set of n horses, consider the subsets $A_1 = \{x_1, \cdots, x_{n-1}\}$ and $A_2 = \{x_2, \cdots, x_n\}$. Since each of A_1 and A_2 contain $n-1$ horses, all horses in A_1 must be the same color and all horses in A_2 must be the same color. And these sets overlap, so in fact all horses in A must be the same color. Therefore there is no horse of a different color!

   Explain why this is not a valid inductive proof.

   (b) Consider the following inductive "proof" that all natural numbers are interesting. To begin with, the first case $n = 1$ is clearly satisfied, since 1 is a very interesting number. Next, suppose there are uninteresting natural numbers. Then there must be a smallest such number, call it n. But n is the smallest uninteresting natural number, which is clearly an interesting thing to be! Therefore there aren't any uninteresting natural numbers.

   Explain why this isn't a valid mathematical proof.