Math 227A: Suggested Exercises for Week 5

1. Hatcher 4.3.1, 4.3.3, 4.3.4, 4.3.5, 4.3.17, 4.3.18, 2.3.2, 2.3.4.

2. A reduced homology theory is constructed from a spectrum $K_n$ by associating to a space $X$ the stable homotopy groups $h_i(X) = \pi_{i+n}(X \wedge K_n)$. (To get an unreduced homology theory, we use $X_+$, the space $X$ together with a disjoint basepoint.)

- Check this satisfies the axioms for homology (page 160 in Hatcher).
- The framed bordism groups $\Omega^{fr}_n(M)$ of a manifold $M$ are the groups of framed $n$-dimensional submanifolds up to framed cobordism we considered in class. Show, using $K_n = S^n$, that these groups form an unreduced homology theory. (Hint: you will want to somewhat modify the map into the sphere we constructed in class for a framed submanifold.)