Math 131B-1: Homework 8

Due: March 7, 2014

1. Read Tao Sections 16.1-4.

2. Do Tao exercises 15.7.10, 16.2.2.

3. Do Tao exercise 16.2.3 [Hint: It’s important to pick a function \( g \) for which \( ||g||_\infty \neq \int_0^1 g \). Try working with \( g(x) = x^2 - x \) on \([0, 1]\) and \( g \) is extended periodically to the rest of the real line.]

4. Do Tao exercise 16.2.6. [Hint: Remember that a continuous 1-periodic function \( f \) is the same as a continuous function on \([0, 1]\) with \( f(0) = f(1) \). So you only have to define all your sequences on the interval in this problem.]

5. Prove Pythagoras’ Identity: If \( < f, g > = 0 \), then \( ||f + g||_2^2 = ||f||_2^2 + ||g||_2^2 \).

6. Prove that the convolution \( f \ast g \) of two continuous \( \mathbb{Z} \)-periodic function is continuous. [Hint: We know \( |f(x)| < M \) for some \( M > 0 \). So start by deciding that \( |f \ast g(x) - f \ast g(x')| = |\int_0^1 f(y)g(x - y)dy - \int_0^1 f(y)g(x' - y)dy| \leq M \int_0^1 (g(x - y) - g(x' - y))dx \). Now use uniform continuity of \( g \).]

[Note: This may seem short, but two of the problems above have several parts.]