Math 131B-1: Homework 4

Due: February 3, 2014

1. Read Apostol Sections 4.8-9, 4.11-13, 4.15-17, 4.19-20. [Most of these are short.]

2. Do problems 4.21, 4.25, 4.28, 4.33, 4.38, 4.39 in Apostol.

3. We say that a subset $S$ of a metric space $M$ is *dense* if every open set in $M$ contains a point of $S$.

   - Prove that if $S$ is dense in $M$, every point of $M$ is the limit of a sequence of points in $S$.

   - Prove that if $f : (M, d_M) \to (T, d_T)$ and $g : (M, d_M) \to (T, d_T)$ are two continuous functions from $M$ to a metric space $(T, d_T)$, and $f(s) = g(s)$ for all $s \in S$, then $f = g$ on $M$.

4. Let $f : X \to \mathbb{R}^n$ be a function such that $f(x) = (f_1(x), \ldots, f_n(x))$. Show that $f$ is continuous if and only if each function $f_i : X \to \mathbb{R}$ is continuous.