Math 131B-1: Optional “Homework” 10

1. Do Apostol 12.9, 12.12, 12.14.

2. Let \( f : \mathbb{R}^2 \to \mathbb{R} \) be given by
   \[
   f(x, y) = \begin{cases} 
   \frac{x^3}{x^2 + y^2} & (x, y) \neq (0, 0) \\
   0 & (x, y) = (0, 0)
   \end{cases}
   \]
   Show that all the directional derivatives of \( f \) exist at \((0, 0)\), but \( f \) is not differentiable.

3. Show that if \( f : \mathbb{R}^n \to \mathbb{R}^m \) has \( f'(x) = 0 \) for all \( x \), then \( f \) is constant. (Hint: Two points determine a line, and you can take a directional derivative along any line away from a point.)

4. Try to find a function \( f : \mathbb{R}^2 \to \mathbb{R} \) whose mixed partial derivatives \( D_{2,1}f \) and \( D_{1,2}f \) exist and are not equal. (Hint: By Clairaut’s theorem, what properties can’t these partial derivatives have?)