Math 131B-2: Homework 9

Due: June 6, 2014

1. Read Tao Sections 16.3-5.

2. Prove Pythagoras’ Identity: If \( <f, g> = 0 \), then \( \|f + g\|^2 = \|f\|^2 + \|g\|^2 \).

3. Prove that the convolution \( f \ast g \) of two continuous \( \mathbb{Z} \)-periodic function is continuous. Hint: You will need to use that \( f \) is bounded and \( g \) is uniformly continuous.

4. Do Tao problem 16.2.3. Hint: You can’t do this problem with a single function \( g \); if you try to, you will sometimes get negative values of \( c \) and \( d \) when you solve. Instead, you need to be able to produce functions \( g \) with \( \sup_{[0,1]} g = k \) and \( \int_0^1 g = \ell \) such that \( kA^2 - \ell B^2 > 0 \).

5. Do Tao problems 16.2.6, 16.5.1, 16.5.2, and 16.5.4. Note the existence of a typo in 16.5.4: It should say \( \hat{f}'(n) = 2\pi i n \hat{f}(n) \).

Caveat: This assignment is not as short as it looks. Several of the problems above have multiple parts.