Math 131A-1: Homework 4

Due: January 30, 2015

1. Read Sections 10-11 in Ross.

2. Do problems 9.10, 9.12, 9.14, 10.6, 10.7, and 10.10 in Ross. [You may assume the result of 9.13 in writing up 9.14, but understanding this exercise on your own time is also recommended.]

3. We say a subset $S$ of $\mathbb{R}$ is closed if whenever a sequence $(s_n)$ of numbers in $S$ converges to some $s$, the limit $s$ is also in $S$.
   
   (a) Use exercise (8.9) from last week to show that the interval $[a, b]$ is closed for any real $a < b$.
   
   (b) Give an example of a closed unbounded set in $\mathbb{R}$ (other than $\mathbb{R}$ itself).
   
   (c) Suppose $S \subset \mathbb{R}$ is closed and bounded above. Use exercise (10.7) above to show that $S$ has a maximum.

4. Let $(s_n)$ be the sequence $s_n = 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n}$.

   (a) Show that $s_n$ is increasing and bounded above by 2. [Hint: Multiply $s_n$ by $2^n$, then use an induction formula for $2^n + 2^{n-1} + \cdots + 2^2 + 2 + 1$ proved in the second lecture. Later in the course we will see a general formula for this sort of sum.]

   (b) Show that $s_{n+1} = \frac{1}{2}s_n + 1$, and conclude that $\lim s_n = 2$.

   (c) Let $t_n = 1 + 2 + 4 + \cdots + 2^n$. Observe that $t_{n+1} = 2t_n + 1$. Why can’t we conclude that $\lim t_n = -1$?

      Note that this addresses one of our motivational questions from the first lecture!

5. The sample midterm isn’t technically homework, but don’t forget it exists.

   NB: For your planning purposes, note that 10.6 is the only question above that uses material which is not fair game for the midterm.