Math 131A-1: Homework 9

Due: May 27, 2016

1. Read Sections 31-32 in Ross.

2. Do problems 29.2, 29.4, 29.5, 29.13, 29.16, 29.18, 23.1(a),(c),(e),(g), 23.5, and 31.1 in Ross.

3. The five constants. Recall that the imaginary number \( i \) satisfies the property that \( i^2 = -1 \). Assume that the power series expansions about zero we have computed for \( e^x \), \( \sin x \), and \( \cos x \) are valid on complex numbers as well as real numbers. (This is true, but we won’t prove it in this class.)

   • (a) What are \( i^3 \) and \( i^4 \)? In general, what can you say about \( i^{4k+j} \)?
   • (b) Use the power series expansions for \( e^x \), \( \sin x \), and \( \cos x \) to show that \( e^{ix} = \cos x + i \sin x \) for all \( x \in \mathbb{R} \).
   • (c) Put \( x = \pi \) into the equation from part to prove that \( e^{i\pi} + 1 = 0 \). This gives a relationship between our five most basic analytical constants.