1. Read Sections 19-20 in Ross.

2. Do problems 17.2, 17.3 (a),(c), 17.10, 17.12, 18.4, 18.7, 18.10, 19.1(a),(c),(f),(g), 19.2(b), 19.4 in Ross.

3. The stars over Babylon. For each rational number \( r \in (0, 1] \), write \( r = \frac{p}{q} \) where \( p, q \in \mathbb{N} \) are natural numbers with no common factors. Then consider the following function on \([0, 1]\):

\[
f(x) = \begin{cases} 
\frac{1}{q} & x = \frac{p}{q} \text{ is rational} \\
0 & x \text{ is irrational.} 
\end{cases}
\]

We claim that \( f \) is discontinuous at every rational number in \((0, 1]\) and continuous at every rational.

- Discontinuity at each rational. Let \( x_0 \in (0, 1] \) such that \( x_0 \) is rational. For \( n \in \mathbb{N} \), pick \( x_n \) an irrational in \((x_0 - \frac{1}{n}, x_0) \cap (0, 1]\). Use this sequence to show \( f \) is discontinuous at \( x_0 \).

- Continuity at each irrational. Let \( x_0 \in (0, 1] \) such that \( x_0 \) is irrational. Let \( N \) be a natural number. Let

\[
\delta_N = \min \{|x_0 - \frac{i}{n}| : 0 \leq i \leq n \leq N, \ i, n \in \mathbb{N}\}.
\]

Observe that because \( x_0 \neq \frac{i}{n} \) for any \( \frac{i}{n} \), \( \delta_N > 0 \). Prove that for \( x \in (0, 1] \), if \( |x - x_0| < \delta_N \), then \( |f(x) - f(x_0)| < \frac{1}{N} \). Conclude that \( f \) is continuous at \( x_0 \).

This example helps demonstrate that our intuition for what continuity should “look like” on a graph is in general insufficiently subtle.