Math 1, Lecture 3  
Precalculus

Sample Midterm 1

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books, notes, or calculators. Partial credit will be given for progress toward correct solutions.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: Solutions

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Problem 1.

Let $P(t)$ be a function which gives the population of town in years after 2014. The population of the town is increasing. Answer each of the questions in a sentence.

(a) [3pts.] What does $P(5) = 32,200$ mean?

(b) [3pts.] What does $P^{-1}(40,000)$ represent?

(c) [4pts.] You learn that $P(t)$ is concave down. What does that tell you about the town's population growth?

9. The population in 2019 (five years after 2014) is 32,200 people.

6. $P^{-1}(40,000)$ is the number of years after 2014 it takes for the population to reach 40,000 people.

6. The rate at which the population increases is decreasing, so the town's growth is slowing down.
Problem 2.
Let \( f(x) \) be the function defined as follows:

\[
f(x) = \begin{cases} 
-5 & \text{if } x \leq -1 \\
2x - 3 & \text{if } -1 < x < 2 \\
x^2 - 4 & \text{if } 2 \leq x 
\end{cases}
\]

(a) [5pts.] Sketch a graph of \( f \).
(b) [5pts.] Determine the domain of \( h(x) = \sqrt{f(x)} \).

\[\sqrt{x} \text{ can only be evaluated on } x \geq 0, \text{ so the domain of } h(x) \text{ is } [-3, \infty).\]
Problem 3.

(a) [4pts.] Sketch a graph of the function $g(x)$ which is equal to the function $f(x) = x^2$ compressed horizontally by a factor of $\frac{1}{2}$ and reflected across the x-axis.

(b) [4pts.] Give a formula for the function $g(x)$ you drew in part (a).

(c) [2pts.] Decide whether $g(x)$ is even, odd, or neither. Justify your answer.

\[ g(x) = -(2x)^2 = -4x^2 \]

Even. $g(-x) = -4(-x)^2 = -4x^2 = g(x)$. Equivalently, the graph is preserved by horizontal reflection.
Problem 4.

Find a formula for each of the following lines.

(a) [5pts.] The line parallel to the line \( h(t) = -5t - 3 \) and passing through the point \((2, -12)\).

(b) [5pts.] The line describing how much money you have left after arriving at a fair with fifty dollars and riding \( n \) rides that cost four dollars each.

\[ m = -5 \quad y = -5x + b \]

\[-12 = h(2) = -5(2) + b \]

\[-12 = -10 + b \]

\[-2 = b \]

\[ y = -5x - 2 \]

\[ D(n) = \text{dollars left} \]

\[ D(0) = 50 \]

\[ m = -4 \]

\[ D(n) = 50 - 4n \]
Problem 5.
Let \( f(x) = 3|5 - x| \).

(a) [5pts.] Solve the absolute value inequality \( f(x) \leq 6 \).
(b) [5pts.] Find any horizontal and vertical intercepts of \( f \).

\[ 3|5 - x| \leq 6 \]
\[ |5 - x| \leq 2 \]
\[ -2 \leq 5 - x \leq 2 \]
\[ -7 \leq -x \leq -3 \]
\[ 7 \geq x \geq 3 \]
\[ [3, 7] \]

\( \text{Horizontal} \)

\[ 0 = 3|5 - x| \]
\[ 0 = 15 - x \]
\[ 5 = x \]

\( \text{Vertical} \)

\[ y = f(0) = 3|5 - 0| = 15 \]
\[ y = 15 \]