MTH 996: Exercises from Week 1

1. Prove that the only three-manifolds with genus one Heegaard decompositions are lens spaces.

2. Compute the first homology of the three-manifold given by the Heegaard diagram shown in Figure 2 of [OSz06].

3. Decide which surgery on 3_1 is given by replacing the meridional β-curve on the pringle-chip diagram drawn in class with a copy of the knot.

4. Prove that $\mathrm{Sym}^g(S^1)$ is homotopy equivalent to S^1 for any g.

5. Prove that if Σ is a surface of genus g, the homotopy type of $\mathrm{Sym}^g(\Sigma - \{z\})$ is a skeleton of an n-torus with the usual product CW decomposition.

References

[OSz06] Peter S. Ozsváth and Zoltán Szabó, *An introduction to Heegaard Floer homology*, Floer homology, gauge theory, and low-dimensional topology, Clay Math. Proc., vol. 5, Amer. Math. Soc., Providence, RI, 2006, pp. 3–27.