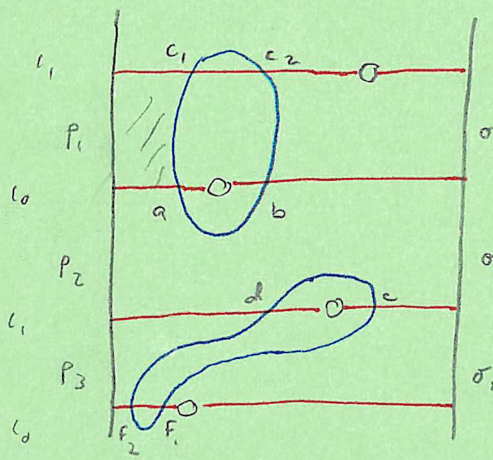
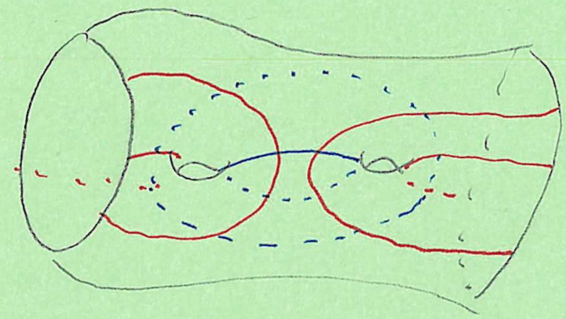
# Lecture 22

**Last Time** $CFK^{\infty}(K) \rightsquigarrow \widehat{CFD}(S^3 - K)$

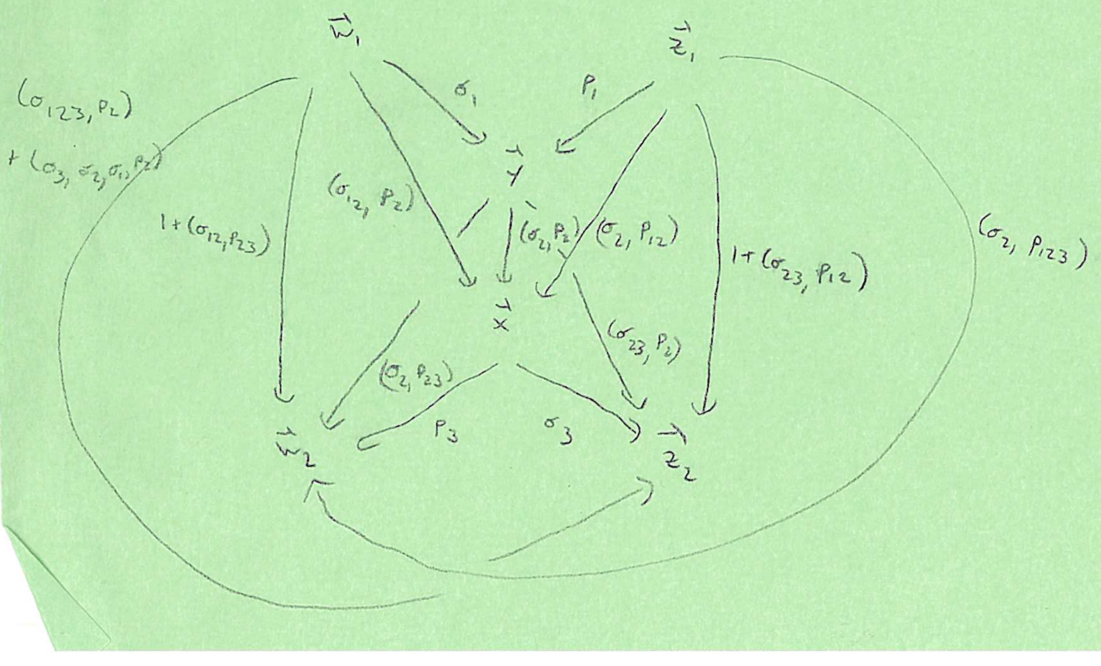
Splicing two knot complements: We can replace a $\widehat{CFD}$ w/ $\widehat{CFA}$ via tensoring with $\widehat{CFAA}(Id)$.



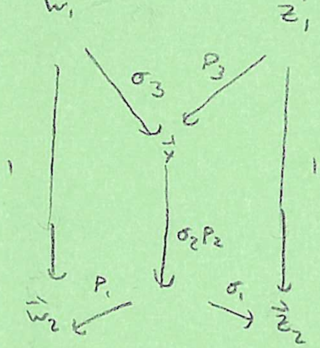
A type AA bimodule has multiplications in on either side

$\vec{y} = ae$ $\quad \vec{x} = bd$ $\quad \vec{w}_1 = bf_1$ $\quad \vec{w}_2 = bf_2$ $\quad \vec{z}_1 = c_1 e$ $\quad \vec{z}_2 = c_2 e$



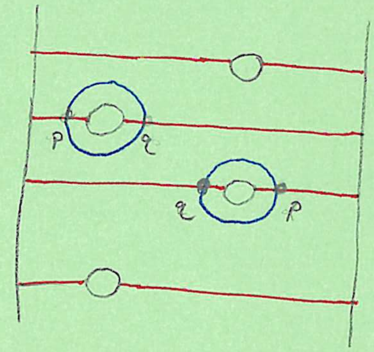
## Example $\widehat{CFDA}(\mathrm{Id})$, $\widehat{CFDD}(\mathrm{Id})$

### $\widehat{CFDD}(\mathrm{Id})$

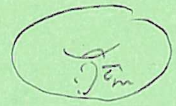


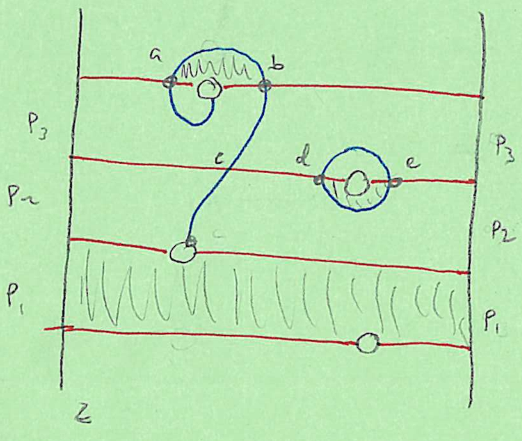

$\rightsquigarrow$

$$\begin{tikzcd} \vec p \arrow[r, bend left, "\rho_1\sigma_3 + \rho_3\sigma_1 + \rho_{123}\sigma_{123}"] & \vec q \arrow[l, bend left, "\rho_2\sigma_2"] \end{tikzcd}$$




## Example A Dehn twist




$\vec p = ae$

$\vec q = bd$

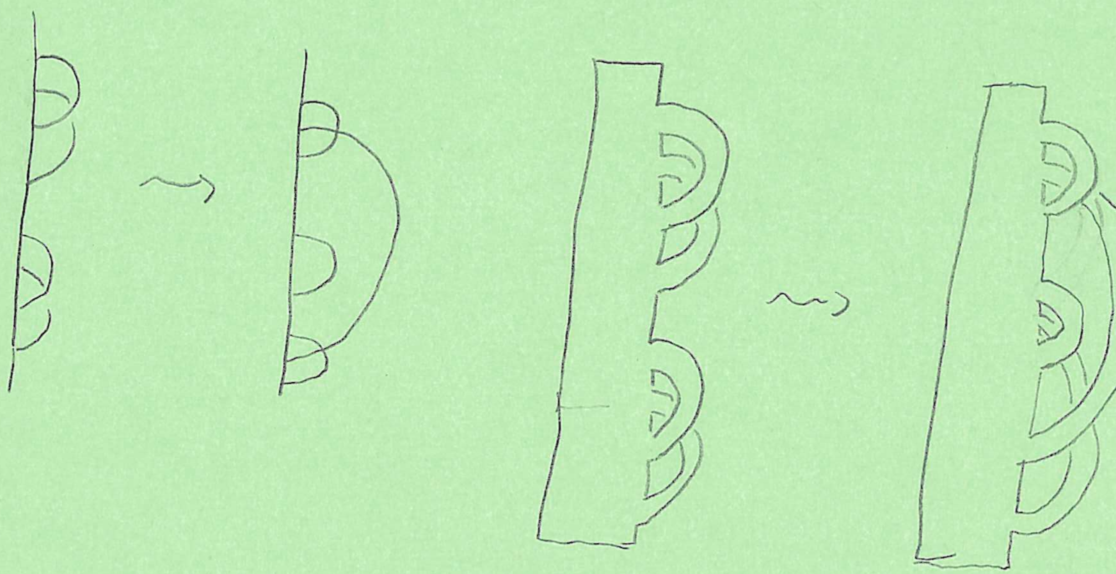
$\vec r = ce$

$\vec p \xrightarrow{\rho_1 \otimes \rho_1} \vec q$

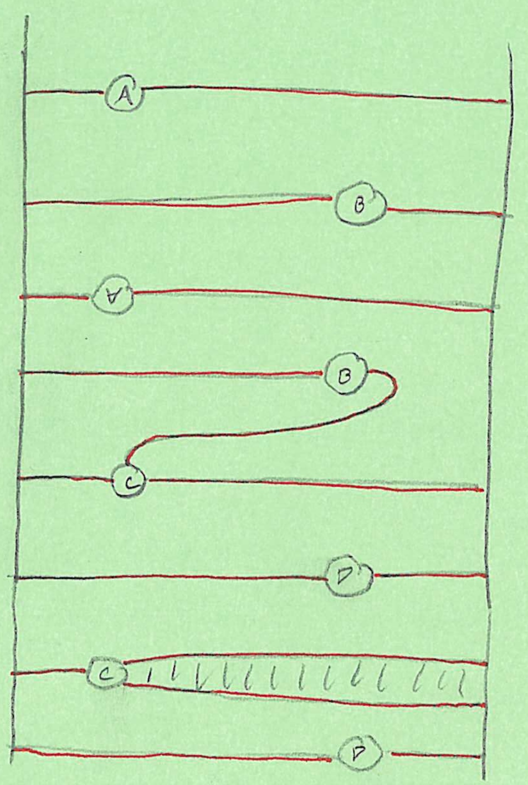
is shaded

$$\widehat{CFDA}(\tau_m) = \begin{tikzcd}[row sep=huge] \vec p \arrow[rr, "\rho_1\otimes\rho_1 + \rho_{123}\otimes\rho_{123} + \rho_3\otimes(\rho_3, \rho_{23})"] \arrow[dr, bend left, "\rho_{123}\otimes\rho_{12} + \rho_3\otimes(\rho_3, \rho_2)"] & & \vec q \arrow[loop right, "\rho_{23}\otimes\rho_{23}"] \arrow[dl, bend left, "\rho_{23}\otimes\rho_2"] \\ & \vec r \arrow[ul, bend left, "\rho_2\otimes 1"] \arrow[ur, bend left, "1\otimes\rho_3"] & \end{tikzcd}$$

- General computations for the Humphries generators are fairly difficult.
- Instead we factor mapping class group elements into a arcslides



Exercise Arcslides generate the mapping class group.



This fairly self-evidently can be written down in a standard format.

For computational purposes:

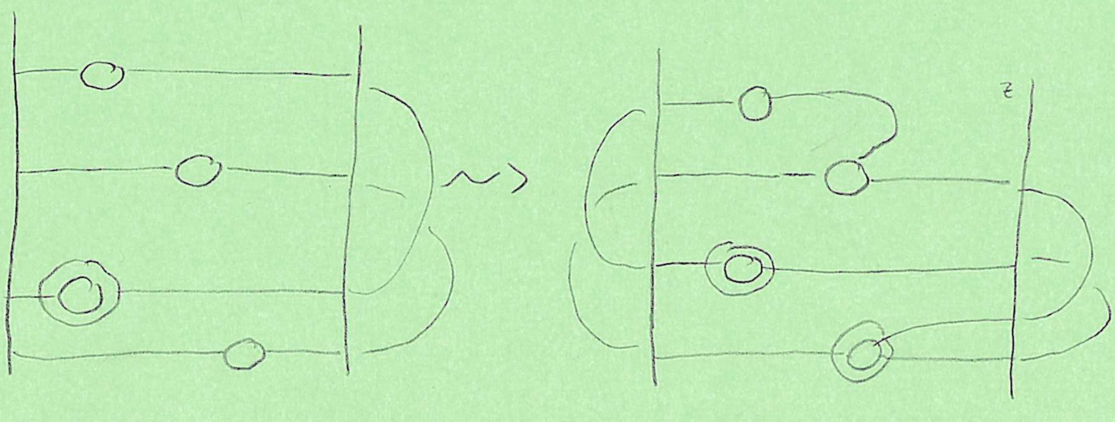
$$\widehat{CF}(Y) \simeq \mathrm{Mor}(\widehat{CFD}(-Y_1), \widehat{CFD}(Y_2))$$

$$\widehat{CF}(Y) \simeq \mathrm{Mor}(\widehat{CFD}(H_K, \phi_0), \mathrm{Mor}(\widehat{CFDD}(-M_{\tilde{\psi}}), \widehat{CFD}(H_K, \phi_0)))$$

$$\widehat{CF}(Y) \simeq \mathrm{Mor}(\widehat{CFD}(H_K, \phi_0), \mathrm{Mor}(\widehat{CFDD}(-M_{\psi_{i_n}}), \mathrm{Mor}(-, \dots)))$$

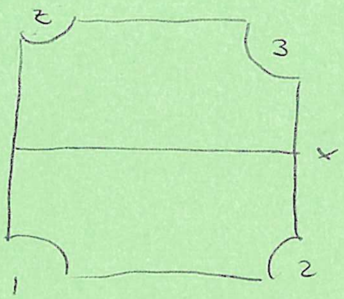
Factoring a Dehn twist into arcslides

Note that an arcslide on $T^2$ goes from $\mathcal{Z}$ to itself:



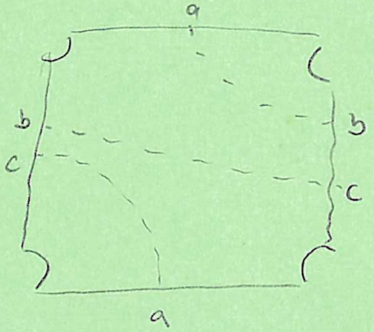
Example

$\mathcal{H}_0$




$\partial x = \rho_{12} x$

$\mathcal{H}_1$




$\partial a = \rho_{123} b$

$\partial b = \rho_{23} c$

$\partial c = \rho_2 a$

$\mathrm{Mor}(\widehat{CFD}(\mathcal{H}_1), \widehat{CFD}(\mathcal{H}_0))$

$a \mapsto x$

$a \mapsto \rho_{12} x$

$b \mapsto \rho_2 x$

$c \mapsto \rho_2 x$

$\partial_{\mathrm{Mor}}(f) = \partial_{P_2} \circ f + f \circ \partial_{P_1}$

$\partial(a \mapsto x) = (a \mapsto \rho_{12} x) + (c \mapsto \rho_2 x)$

$\partial(a \mapsto \rho_{12} x) = 0 \qquad \partial(c \mapsto \rho_2 x) = 0$

$\partial(b \mapsto \rho_2 x) = 0$

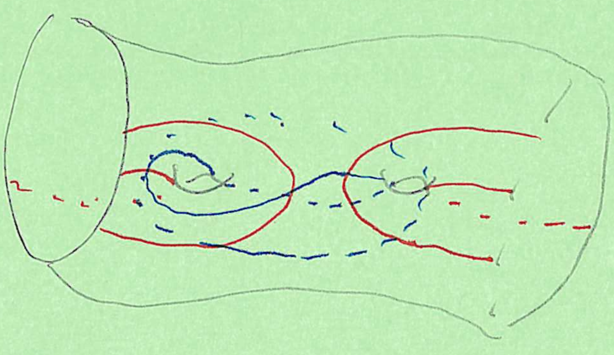
$$\begin{array}{ccc} & a \mapsto x & \\ \swarrow & & \searrow \\ a \mapsto \rho_{12} x & & c \mapsto \rho_3 x \end{array} \qquad b \mapsto \rho_2 x$$

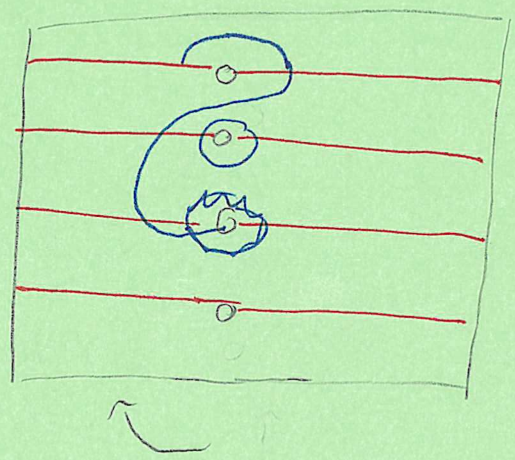
We can take $B(F) = \mathcal{A}(\mathcal{Z}, 1-n)$. Then we can look at the category of left $\mathcal{A}(\mathcal{Z}, 1)$ modules.

- $N(\phi) = \widehat{CFDA}(M_\phi, -n+1)$

- Tensoring by $\widehat{CFDA}$ gives an action of the mapping class group of the surface on the derived category of left $\mathcal{A}(\mathcal{Z}, 1)$ modules. (homotopy category of finitely-generated projective modules)
- This action is faithful; i.e. if $\widehat{CFDA}(\phi) \simeq \widehat{CFDA}(\mathbb{I})$ then $\phi \simeq \mathbb{I}$.

Why this algebra? It corresponds to taking a standard identity bimodule and applying $\phi^{-1}$ to the beta curves on the lefthand side.





we want things w/ exactly one generator on this side