MTH 961: Suggested Exercises for Week 8

1. Prove that $H^*(K(\mathbb{Z}_n);\mathbb{Q}) \simeq \mathbb{Q}[x]$ for n even and $\wedge_{\mathbb{Q}}[x]$ for n odd, where x is an element of $H^n(K(\mathbb{Z},n);\mathbb{Q})$.

2. Prove that if a pair (X,Y) is n-connected, and $E \to X$ is a fibration, then $(E,\pi^{-1}(Y))$ is also n-connected. Use this to check directly that the Serre spectral sequence is first-quadrant. (The interesting case is $p > 0$, $q < 0$.)

3. (Carried over from last week.) Use the Serre spectral sequence to give an (overpowered) reproof of the Leray-Hirsch theorem:

   Theorem 0.1. *Let $F \xrightarrow{i} E \xrightarrow{p} B$ be a fibration. Suppose there are classes c_j in $H^*(E;\mathbb{Q})$ whose restrictions $i^*(c_j)$ form a basis for $H^*(F;\mathbb{Q})$. Then $H^*(E;\mathbb{Q}) \simeq H^*(F;\mathbb{Q}) \otimes H^*(B;\mathbb{Q})$ as a vector space.*

4. (This question uses Thursday's material.)

   - Show that $\pi_k(S^n)$ is finite for n odd and $n < k$.
   - Show that $\pi_k(S^n)$ is finite for n even and $n < k < 2n-2$. Show that $\pi_{4m-1}(S^{2m})\otimes\mathbb{Q} \simeq \mathbb{Q}$.