MTH 961: Suggested Exercises for Week 3

1. Do exercises 4.3.3, 4.3.4, 4.3.7, 4.3.10, 4.3.17, 4.3.18, 2.3.2, and 2.3.4 in Hatcher.

2. A reduced homology theory is constructed from a spectrum K_n by associating to a space X the stable homotopy groups $h_i(X) = \pi_{i+n}(X \wedge K_n)$. (To get an unreduced homology theory, we use X_+, the space X together with a disjoint basepoint.)

   - Check this satisfies the axioms for homology (page 160 in Hatcher).
   - The framed bordism groups $\Omega_n^{fr}(M)$ of a manifold M are the groups of framed n-dimensional submanifolds up to framed cobordism we considered in class. Show, using $K_n = S^n$, that these groups form an unreduced homology theory. (Hint: you will want to somewhat modify the map into the sphere we constructed in class for a framed submanifold.)