

MTH 961: Suggested Exercises for Week 14

1. Let M be compact and let either $\Lambda = \mathbb{Z}_2$ or $\Lambda = \mathbb{Q}$ and M is oriented. Prove that the map $u''/: H_{n-k}(M; \Lambda) \rightarrow H^k(M; \Lambda)$ given by taking the slant product with the diagonal cohomology class u'' is an isomorphism with inverse given by $(-1)^{nk}$ times the cap product with the fundamental class μ of M , $\cap\mu: H^k(M) \rightarrow H_{n-k}(M)$.
2. Let $M = M^n$ and $A = A^p$ be orientable compact manifolds, and $i: M \hookrightarrow A$ be an embedding. Let $k = p - n$. Prove that the Poincaré duality map $\cap\mu_A: H^k(A) \rightarrow H_n(A)$ maps the cohomology class $u'|_A$ dual to M to $(-1)^{nk}i_*(M)$. [There is a helpful commutative diagram explaining how to use the tubular neighborhood theorem for this problem in Milnor and Stasheff exercise 11C.]
3. Prove that all Stiefel-Whitney classes of a three-manifold are zero. Assuming the results about cobordism stated in class, what does this tell you?
4. Prove that a vector bundle ξ over a CW complex is orientable if and only if $w_1(\xi) = 0$.
5. Using the Wu formula and $\pi_2(V_2(\mathbb{R}^3)) = \pi_2(SO(3)) = 0$, show that every compact orientable three-manifold is parallelizable.