MTH 961: Suggested Exercises for Week 12

1. Show that the orthogonal complement of a subbundle of a vector bundle over a paracompact base is, up to isomorphism, independent of the choice of inner product.

2. A vector $E \to B$ is said to have finite type if there is some $E' \to B$ such that $E \oplus E' \simeq \epsilon^{\oplus n}$. In class we proved that every vector bundle over a compact base has finite type. Prove that the canonical line bundle over $\mathbb{RP}^\infty$ does not have finite type.

3. Prove that if $n + 1 = 2^r m$ with m odd, then there do not exist 2^r vector fields on $\mathbb{RP}^n$ which are nowhere dependent.

4. Show that if $\mathbb{RP}^n$ can be immersed in $\mathbb{R}^{n+1}$, then $n = 2^r - 1$ or $n = 2^r - 2$.

5. Verify that the construction of the Stiefel-Whitney classes given in class using the Leray-Hirsch Theorem satisfies the four axioms for these characteristic classes.

6. Show there is a three-dimensional real vector bundle over S^4 which is not the direct sum of a one-dimensional bundle and a two-dimensional bundle. [You're missing one fact you need, which is that an arbitrary two-dimensional vector bundle over S^4 can be given the structure of a complex line bundle. But otherwise you know enough to produce this example.]