MTH 961: Suggested Exercises for Week 11

1. Determine the canonical line bundle on $\mathbb{RP}^1$ (this has a simple geometric answer).
2. Verify that if $E \to B$ is a vector bundle, and $f, g \colon C \to B$ are homotopic maps, then the pullback bundles $f^*(E)$ and $g^*(E)$ are isomorphic as vector bundles.
3. Let M be a smooth manifold, and consider the diagonal embedding $\Delta \colon M \to M \times M$. Construct the tangent and normal bundles to this embedding, and prove they are isomorphic.
4. Prove that if n is odd, the unit sphere S^n admits a nowhere zero vector field. Furthermore, show that for all n, the normal bundle to $S^n \subset \mathbb{R}^{n+1}$ is trivial.
5. Prove that if S^n admits a vector field which is nowhere zero, the identity map on S^n is homotopic to the antipodal map. Use this to show that if n is even, S^n is not parallelizable.
6. Show that the set of isomorphism classes of line bundles over B forms an abelian group with respect to taking the tensor product of vector bundles.