MTH 961: Suggested Exercises for Week 1

1. Do exercises 4.2.8 and 4.2.9 in Hatcher.
2. If you have not already done any of exercises 4.2.3, 4.2.12, 4.2.22, 4.2.26, 4.2.31-34 in Hatcher, take a look at those exercises. (I assume most of them were already assigned.)
3. Compute the homotopy groups of $\Sigma(S^1 \times S^1)$ (and note that the hypotheses of the suspension theorem are quite necessary).
4. Use the description of the stable homotopy groups of spheres given in class to find $\pi_{n+1}(S^n)$ for $n \geq 3$. Use this to confirm the assertion made in class that excision fails for the pair (S^3, S^2). (Note that it is possible that this description of the stable homotopy groups will not be given until the beginning of next week; I am not currently quite certain of our timing.)

A couple of exercises on previous material that don't appear in Hatcher:

1. Show that if G contains any element of finite order, then $K(G, 1)$ has the homotopy type of an infinite-dimensional CW complex.
2. (Mostly targeted at people who like Heegaard Floer theory)

   Given a space X with the homotopy type of a CW complex, its nth *symmetric product* is the quotient of the product X^n by the action of the symmetric group S_n on the factors. More concretely, a point in $\mathrm{Sym}^n(X)$ is a g-tuple of unordered points in X.

   - What are the homotopy types of $\mathrm{Sym}^n(S^1)$ and $\mathrm{Sym}^n(\mathbb{C})$?
   - Let X be an n-punctured surface of genus g. What are the homotopy groups of $\mathrm{Sym}^{g+n-1}(X)$? (Hint: There is a relationship between symmetric products of wedge sums and symmetric products of ordinary product spaces.)
   - Restrict to the case that X is a sphere. Let $\{\alpha_1, \cdots, \alpha_n\}$ be a set of nonintersecting simple closed curves such that each component of $X - \{\alpha_1, \cdots, \alpha_n\}$ contains a single puncture. There is a torus $T_\alpha = \alpha_1 \times \cdots \times \alpha_n$ contained in $\mathrm{Sym}^{g+n-1}(X)$. Show (without explicitly writing down a map) that $\mathbb{T}_\alpha$ is a deformation retract of $\mathrm{Sym}^{n-1}(X)$.