MTH 961: Algebraic Topology II

Instructor: Kristen Hendricks

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Office: D320 Wells

Course Location and Time: T Th 10:20 – 11:40, A222 Wells Hall

Office Hours: T Th 3-4, or by appointment

Prerequisites: Math 960 or equivalent. (Equivalent in this case means having read and understood Hatcher's *Algebraic Topology*, Chapters 1-3,4.1-2, not including the special topics, and with the exception of the Freudenthal Suspension Theorem.)

Topics: This is a third semester course in algebraic topology. We will cover connections between homotopy theory and cohomology, cohomology theories, stable homotopy theory and cobordism, basic obstruction theory, K-theory and characteristic classes, and spectral sequences, possibly not in precisely that order.

Textbooks: Main texts for this course will include:

- A. Hatcher, Algebraic Topology. (Available at Allen Hatcher's website.)
- J. Milnor and J. Stasheff, *Characteristic Classes*.
- A. Hatcher, *Vector Bundles and K-Theory*. (Available at Allen Hatcher's website.)
- A. Hatcher, *Spectral Sequences*. (Available at Allen Hatcher's website.)

Other texts that may be of interest include:

- R. Bott and L. Tu, *Differential Forms in Algebraic Topology*. (Available to MSU affiliates through SpringerLink.)
- J. Milnor, Topology from the Differentiable Viewpoint.
- J. Davis and P. Kirk, *Lecture Notes in Algebraic Topology*. (Available on Jim Davis's website.)

Please let me know if you encounter any difficulties in getting access to copies of these texts.

Homework: There will be weekly suggested exercises. These will not be collected, but you are encouraged to come talk to me about them in office hours. You will get noticeably more out of the course if you complete them promptly.

Motivation: Here are a few questions that can be either addressed or helpfully rephrased using the techniques of this course:

Given a closed manifold M, a framed submanifold N is an embedded submanifold together with a smoothly varying basis for the normal bundle at each point in the set. What can one say about the set of such submanifolds, up to framed cobordism in M? How does this change if the framing condition is removed?

Given a smooth manifold M of dimension m, what are the smallest numbers n and k such that M may be immersed into n-dimensional Euclidean space and embedded into k-dimensional Euclidean space?

For what values of n is there a bilinear multiplication on Rⁿ without zero divisors? We are aware of n=1 (the real numbers), n=2 (the complex numbers), n=4 (the quaternions), and n=8 (the octonions). Are there any others?

How can one distinguish between smooth manifolds that are homeomorphic but not diffeomorphic?