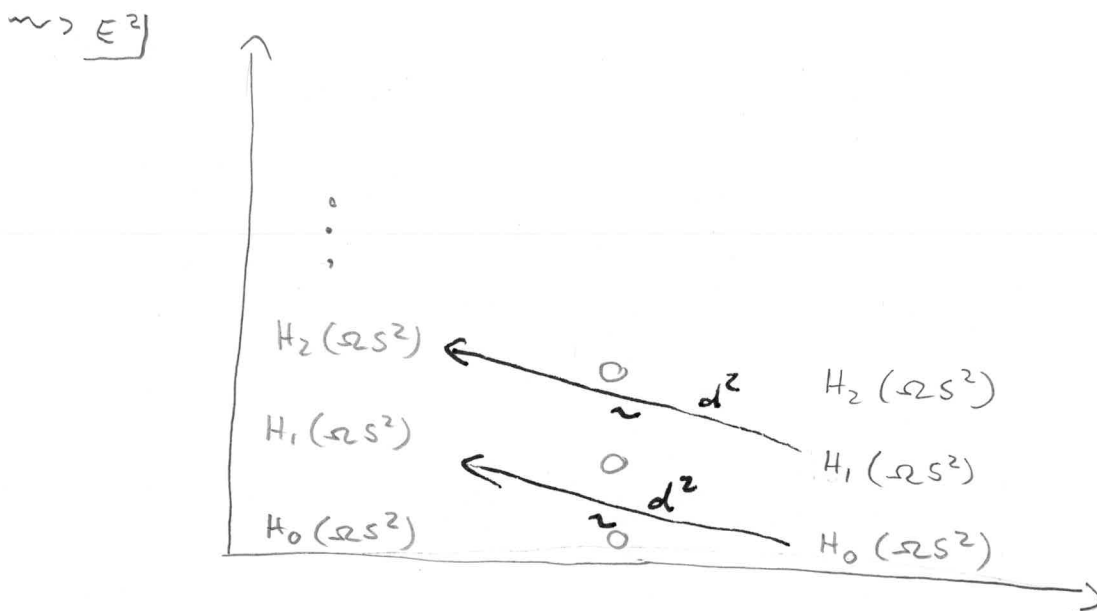
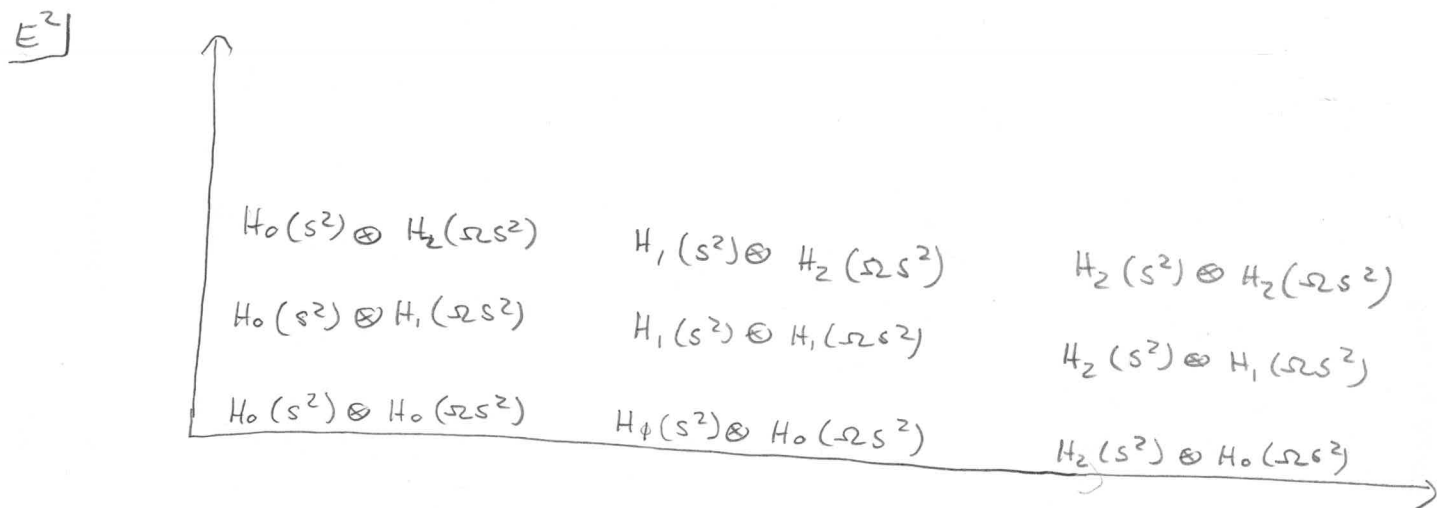


Example What is $H_*(\Omega S^2)$? $H_*(\Omega S^n)$?

Consider $\Omega S^2 \rightarrow \mathbb{P}S^2$



Serre spectral sequence



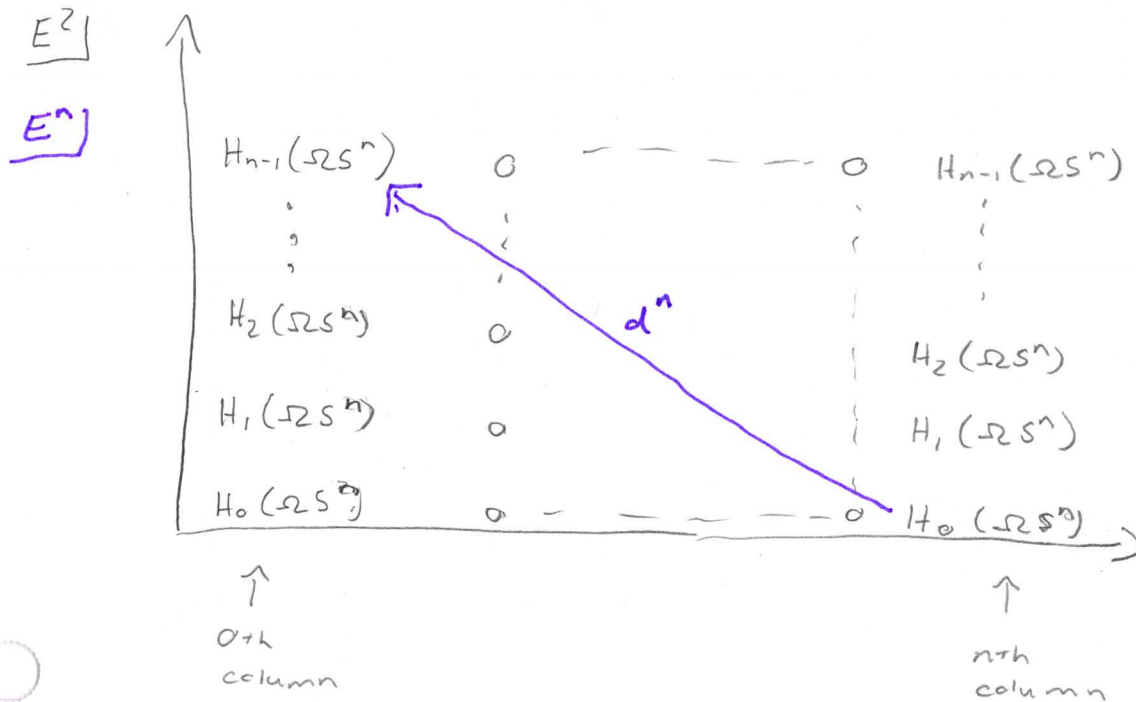
$\leadsto H_0(\Omega S^2) = \mathbb{K}$

$H_1(\Omega S^2) = \mathbb{K}$

$H_{i+1}(\Omega S^2) = H_i(\Omega S^2) = \mathbb{K}$

Now consider $\Omega S^n \longrightarrow \mathcal{P}S^n$

\downarrow
 S^n



No changes until the E^n page, when we have

$$d^n: H_0(\Omega S^n) \xrightarrow{\sim} H_{n-1}(\Omega S^n), \text{ and indeed}$$

$$d_n: H_i(\Omega S^n) \xrightarrow{\sim} H_{i+n-1}(\Omega S^n).$$

Since $H_0(\Omega S^n) = \mathbb{K}$, and $H_i(\Omega S^n) = 0$ for $1 \leq i \leq n-2$ by $(n-1)$ -connectedness or from looking at the spectral sequence, we have

$$H_i(\Omega S^n) = \begin{cases} \mathbb{K} & \text{if } i \text{ is a multiple of } n-1 \\ 0 & \text{otherwise} \end{cases}$$