Homework 9 Solutions

MTH 327H

4.
- $M = (1, 2)$ is connected and $S = \mathbb{Q}$ is not, so no such surjection exists.
- $f\colon (1, 2) \to [0, 1]$ given by

$$f(x) = \begin{cases} 0 & x < \frac{5}{4} \\ 2x - \frac{5}{2} & \frac{5}{4} \leqslant x \leqslant \frac{7}{4} \\ 1 & x > \frac{7}{4} \end{cases}$$

  is a suitable continuous surjection.
- $M = [0, 1]$ is compact and $S = [1, 2)$ is not, so no such surjection exists.
- $M = [0, 1]$ is compact. I claim $S = \mathcal{C}([0, 1], \mathbb{R})$ is not. For consider the sequence of functions $f_n(x) = n$ in S. This sequence (f_n) has no convergent subsequence in S, since its elements become arbitrarily far away from any function in S. So since M is compact and S is not, we see that no continuous surjection from M to S exists.
- The function $f(x) = (\cos x, \sin x)$ is a satisfactory continuous surjection.
- Suppose that $f\colon (1, 2) \cup (3, 4) \to \{0, 1, 2\}$ is continuous. Then $f((1, 2))$ is connected, and therefore f must be constant on $(1, 2)$, mapping all of $(1, 2)$ to 0, 1, or 2. Similarly f must be constant on $(3, 4)$. Ergo f cannot hit all three elements of $\{0, 1, 2\}$, and no continuous surjection from M to S exists.
- $f(x, y) = \arctan(\pi x - \frac{\pi}{2})$ is a suitable continuous surjection from $(0, 1) \times (0, 1)$ to $\mathbb{R}$.
- M is the unit circle on the complex plane and is in particular a compact set; S is not closed and therefore not compact. So no continuous surjection from M to S exists.