MTH 327H: Homework 2

Due: September 14, 2018

1. Office hours the third week of classes are M 11:30-12:30, W 3-4, and Th 9-10.
2. Read Rudin Sections 1.1-22 and 2.1-14.
3. Do exercises 3 and 9 from the "Equivalence Relations" supplement and exercise 15 from the "Induction" supplement.
4. Prove by induction that $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$.
5. Let $\sim$ be an equivalence relation on a set S. Prove that if $[a]$ and $[b]$ are two equivalence classes, either $[a] = [b]$ or $[a] \cap [b] = \emptyset$.
6. Prove using the Peano axioms and the definitions of $+$ and $\times$ on $\mathbb{N}$ given in class that for $a, b \in \mathbb{N}$, $a \times b = b \times a$. [You can use that we already checked that $a \times 1 = 1 \times a$.]
7. Prove that the operation of multiplication on $\mathbb{Q}$ given in class is well-defined.
8. We can construct $\mathbb{Z}$ from $\mathbb{N}$ via an equivalence relation in the following way: let $S = \{(a, b) : a, b \in \mathbb{N}\}$, and consider the relationship $(a, b) \sim (c, d)$ if $a + d = c + b$.
   - Prove that $\sim$ is an equivalence relation.
   - Let $[(a, b)] + [(c, d)] = [(a + c, b + d)]$ and $[(a, b)] \times [(c, d)] = [(ac + bd, bc + ad)]$ be the operations of addition and multiplication on $\mathbb{Z} = S/\sim$. Prove that these operations are well-defined.
   - Prove that this construction of $\mathbb{Z}$ satisfies all of the axioms in the list on pages 5-6 of Rudin *except* (M5). (This implies that $\mathbb{Z}$ is a unital commutative ring rather than a field.)
9. Suppose that A and B are sets, and denote by A^B the set of all maps $f \colon B \to A$. Construct a bijection between $A^{B \times C}$ and $(A^B)^C$.
10. Prove that if A and B are nonempty finite sets, then $|A^B| = |A|^{|B|}$.