MTH 320, Section 003
Analysis

Sample Midterm 2

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

(a) [5pts.] Let $f\colon \mathbb{R} \to \mathbb{R}$. What does it mean to say that f is continuous at x_0?

(b) [5pts.] A set S is said to be *dense* in $\mathbb{R}$ if every open interval contains a point in S. (For example, both the rationals and the irrationals are dense in $\mathbb{R}$.) Suppose S is dense in $\mathbb{R}$, $f, g\colon \mathbb{R} \to \mathbb{R}$ are continuous on $\mathbb{R}$, and $f(s) = g(s)$ for every $s \in S$. Prove that $f(x) = g(x)$ for every $x \in \mathbb{R}$.

Problem 2.

For each of the following, either give an example of a power series with the given properties, or prove that one cannot exist.

(a) [3pts.] A power series with interval of convergence $(0, 2]$.

(b) [4pts.] A power series which converges uniformly on its interval of convergence.

(c) [3pts.] A power series with interval of convergence $[2, 3]$.

Problem 3.

(a) [5pts.] Let $f : \mathbb{R} \to \mathbb{R}$. What does it mean to say that f is differentiable at a?

(b) [5pts.] Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two real-valued functions such that $g(a) \neq 0$, and both f and g are differentiable at a. Prove that $\frac{f}{g}$ is differentiable at a.

Problem 4.

Let (f_n) be a sequence of real-valued functions.

(a) [5pts.] What does it mean for (f_n) to converge uniformly to f on a domain $S \subset \mathbb{R}$?

(b) [5pts.] Suppose that (f_n) is uniformly continuous on S, and $f_n \to f$ uniformly on S. Prove that f is uniformly continuous on S.

Problem 5.

Let (a_n) be a sequence of positive numbers such that $\lim a_n = 0$.

(a) [5pts.] Give an example to show that $\sum a_n$ need not converge.

(b) [5pts.] Prove that there exists a subsequence (a_{n_k}) of (a_n) such that $\sum_{k=1}^{\infty} a_{n_k}$ converges.

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on this page unless you indicate *very clearly* that this is the case on the page of the corresponding problem.