

**MTH 320, Section 003**  
**Analysis**

**Sample Midterm 1**

**Instructions:** You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: \_\_\_\_\_

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

**Problem 1.**

- (a) [5pts.] Let  $F$  be a field, and  $\leq$  an order relation. List the axioms that  $\leq$  must satisfy.
- (b) [5pts.] Let  $F = \{0, 1\}$ .  $F$  can be given the structure of a field with addition and multiplication

$$0 + 0 = 0$$

$$1 + 0 = 1$$

$$0 + 1 = 1$$

$$1 + 1 = 0$$

$$0 \times 0 = 0$$

$$1 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 1 = 1$$

Show that  $F$  cannot be given the structure of an ordered field.

**Problem 2.**

Let  $S \subset \mathbb{R}$  be a nonempty bounded subset of  $\mathbb{R}$ .

- (a) [5pts.] Define the supremum and infimum of  $S$ .
- (b) [5pts.] Let  $S$  and  $T$  be two subsets of  $\mathbb{R}$  which are bounded above. Let  $S \cup T$  be their union, i.e.  $x \in S \cup T$  if and only if  $x \in S$  or  $x \in T$ . Show that  $\sup(S \cup T) = \max\{\sup S, \sup T\}$ .

**Problem 3.**

- (a) [5pts.] Define a Cauchy sequence.
- (b) [5pts.] Prove that if  $(s_n)$  and  $(t_n)$  are Cauchy, then their product  $(s_nt_n)$  is also a Cauchy sequence. (Hint: This is extremely similar to the corresponding proof for convergent sequences.)

**Problem 4.**

Let  $(s_n)$  be a sequence of real numbers.

- (a) [5pts.] Suppose that  $(s_n)$  and  $(t_n)$  are bounded sequences of nonnegative numbers. Prove that  $\limsup(s_n t_n) \leq (\limsup s_n)(\limsup t_n)$ .
- (b) [5pts.] Give an example to show that the inequality in part (a) need not be an equality.

**Problem 5.**

Let  $s_n$  be a sequence defined recursively by  $s_1 = 10$  and  $s_n = \frac{1}{4}(s_{n-1} + 6)$ .

- (a) [5pts.] Show that  $(s_n)$  is decreasing and satisfies  $s_n > 2$  for all  $n$ .
- (b) [5pts.] Does  $(s_n)$  converge? If so, what is the limit? Justify your answer carefully.

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on this page unless you indicate *very clearly* that this is the case on the page of the corresponding problem.