MTH 320, Section 003
Analysis

Midterm 1

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

(a) [5pts.] List the nine defining axioms of a field F.

(b) [5pts.] Suppose you are given a three-element set $F = \{0, 1, a\}$ and told it has the structure of a field; that is, it has an operation $+$ and and operation $\times$ that satisfy the axioms above. Decide what $a \times a$ is. [Hint: Because fields are closed under multiplication, your answer should be one of the three elements listed. It may be helpful to first decide which of the three elements is the multiplicative inverse of a.]

Problem 2.

Let $S \subset \mathbb{R}$ be a nonempty bounded subset of $\mathbb{R}$.

(a) [5pts.] Define the supremum and infimum of S.

(b) [5pts.] Let S and T be nonempty subsets of the $\mathbb{R}$ with the property that $s \leq t$ for all $s \in S$ and $t \in T$. Prove that $\sup S \leq \inf T$. Is it possible for $S \cap T$ to be nonempty?

Problem 3.

(a) [5pts.] What does it mean for a sequence to be bounded?

(b) [5pts.] Let (a_n), (b_n) be sequences of positive real numbers such that $\frac{a_n}{b_n} \to 1$. Prove that if (b_n) is bounded, then (a_n) is also bounded. [Warning: It is *not* necessarily true that $a_n < b_n$ for any particular n.]

Problem 4.

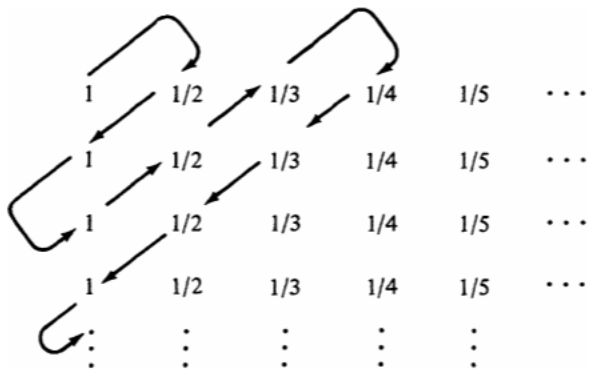
Let (s_n) be a sequence defined recursively by $s_1 = 1$ and $s_{n+1} = [1 - \frac{1}{(n+1)^2}]s_n$ for $n \geq 1$.

(a) [5pts.] Prove, using the recursive definition of (s_n) given above, that (s_n) is a convergent sequence.

(b) [5pts.] Use induction to show that $s_n = \frac{n+1}{2n}$. What is $\lim s_n$?

Problem 5.

Let (s_n) be the sequence of numbers shown in the indicated order.



(a) [5pts.] What is the set S of subsequential limits of (s_n)?

(b) [5pts.] What are $\limsup s_n$ and $\liminf s_n$?

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on this page unless you indicate *very clearly* that this is the case on the page of the corresponding problem.