MTH 320: Homework 2

Due: September 15, 2017

1. Don't forget that the first in-class quiz is Monday, September 18 for the first twenty minutes of lecture. It will be on the material up to the end of lecture on Wednesday, September 13 (this means the relevant material is Sections 1-4 and 7-8).

2. Read Sections 4, 7-8 in Ross.

3. Do problems 3.4, 3.7, and 3.8 in Ross.

4. Do problems 4.1 - 4.4 in Ross for (a), (b), (m), (r), and (w). [Please do not use the answer formats suggested by the textbook; instead use complete sentences and standard capitalization.]

5. Let F be a field; that is, F is a set with two operations $+$ and $\times$ obeying the nine field axioms introduced in class.

   (a) Show that the additive identity 0 postulated by axiom (A3) is unique; that is, show that if there is another element $0'$ satisfying $a + 0' = a$ for all a in F, then $0' = 0$. Show also that for each $a \in F$, the additive inverse $-a$ is unique.

   (b) Show that the multiplicative identity 1 postulated by axiom (M3) is unique, and that for each nonzero $a \in F$, the multiplicative inverse a^{-1} is unique.

6. Recall that the complex numbers $\mathbb{C}$ are the set of all numbers $a + bi$ such that $a, b \in \mathbb{R}$ and i is a number satisfying $i^2 = -1$. The operations of addition and multiplication on $\mathbb{C}$ are as follows:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
$$(a + bi) \times (c + di) = (ac - bd) + (ad + bc)i$$

   (a) Show that $\mathbb{C}$ is a field.

   (b) Show there is no relation $\leq$ on $\mathbb{C}$ which makes $\mathbb{C}$ into an ordered field.

7. Given a set A and a set B, the product $A \times B$ is the set of elements $\{(a, b) : a \in A, b \in B\}$. Prove that, if A and B are countable, then $A \times B$ is countable.

8. Consider the set consisting of all subsets of the natural numbers. Sample elements of this set include $\emptyset$, $\{1, 3, 17\}$, $\{m \in \mathbb{N} : m \text{ is even}\}$, $\mathbb{N}$ itself, and so on. This is called the *power set* $P(\mathbb{N})$ of $\mathbb{N}$. Prove that this set is uncountable.