MTH 310, Section 001
Abstract Algebra I and Number Theory

Sample Midterm 1

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books or notes. Justify all of your answers. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

(a) [5pts.] Use the Euclidean Algorithm to determine $(272, 1479)$.

(b) [5pts.] Prove that for integers a, b, c, we have $((a, b), c) = (a, (b, c))$.

Problem 2.

(a) [5pts.] State the Fundamental Theorem of Arithmetic.

(b) [5pts.] If $p > 3$ is a prime number, prove that $p^2 + 2$ is necessarily composite. [Hint: Consider the possible remainders when p is divided by 3.]

Problem 3.

Consider the ring $\mathbb{Z}_8$.

(a) [5pts.] Give a complete list of subrings of $\mathbb{Z}_8$.

(b) [5pts.] What are the solutions to $x^3 + x^2 + x + 1 = 0$ in $\mathbb{Z}_8$?

Problem 4.

(a) [5pts.] Let R be a ring. What does it mean for an element $a \in R$ to be a zero divisor?

(b) [5pts.] Prove that if ab is a zero divisor in a ring R, then at least one of a and b is a zero divisor.

Problem 5.

Decide whether the following two subsets of $\mathbb{R}$ are subrings of $\mathbb{R}$.

(a) [5pts.] $S = \{a\sqrt{2} : a \in \mathbb{Z}\}$

(b) [5pts.] $T = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on this page unless you indicate *very clearly* that this is the case on the page of the corresponding problem.