Instructions: You have 25 minutes to complete the quiz. There are two problems, worth a total of ten points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ________________________________

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Problem 1.

(a) [2pts.] Let $S$ and $R$ be rings. What does it mean for $f : S \to R$ to be an isomorphism?

(b) [3pts.] Prove that the following pairs of rings are not isomorphic.

- (i) $\mathbb{Z}_6$ and $\mathbb{Z}_{24}$
- (ii) $\mathbb{Z}_{24}$ and $\mathbb{Z}_2 \times \mathbb{Z}_{12}$
- (iii) $\mathbb{Z} \times \mathbb{Z}_3$ and $\mathbb{Z}$
Problem 2.

(a) [2pts.] Let $F$ be a field. State the Division Algorithm for polynomials in $F[x]$.

(b) [3pts.] What are the remainder and quotient when $f(x) = 3x^4 + 2x^3 + 1$ is divided by $g(x) = 2x^2 + 4$ in $\mathbb{Z}_5[x]$?