MTH 310, Section 001
Abstract Algebra I and Number Theory

Quiz 1

Instructions: You have 25 minutes to complete the quiz. There are two problems, worth a total of ten points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	5	
2	5	
Total:	10	

Problem 1.

(a) [2pts.] Define the greatest common divisor (a, b) of two integers a and b.

Solution: The greatest common divisor of a and b is the largest integer d such that $d|a$ and $d|b$.

(b) [3pts.] Let a, b, u, v be integers such that $au + bv = 1$. Prove that $(a, b) = 1$.

Solution: Suppose not. Then there is some integer $d > 1$ such that $d|a$ and $d|b$. Let $a = ds$ and $b = dt$. We have $1 = dsu + dtv = d(su + tv)$, so $d|1$. This is a contradiction. So $(a, b) = 1$.

Problem 2.

(a) [2pts.] What does it mean for an integer p to be prime?

Solution: We say an integer p is prime if $p \neq 0, \pm 1$ and the only divisors of p are ± 1 and $\pm p$.

(b) [3pts.] If $p \geq 5$ and p is a prime, prove that in $\mathbb{Z}_6$ either $[p] = [1]$ or $[p] = [5]$.

Solution: Suppose not, then $[p]$ is one of $[0], [2], [3]$, or $[4]$. If $[p] = [0]$ then $p = 0 + 6k = 6k$ for some k, and p is in particular divisible by 6, and therefore not prime. If $p \geq 5$ and $[p] = [2]$, then $p = 6k + 2$ for some $k > 1$, so p is in particular even and not equal to 2, hence not prime. Similarly if $[p] = [4]$. Finally, if $[p] = [3]$ and $p \geq 5$, we see that $p = 6k + 3 = 3(2k + 1)$ for some $k > 1$, so in particular p is divisible by 3 and not equal to 3, hence not prime. Hence all four cases are ruled out.