MTH 310, Section 001
Abstract Algebra I and Number Theory

Midterm 2

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

(a) [5pts.] What does it mean for two rings R and S to be isomorphic?

(b) [5pts.] Let R be a ring and $R^* \subset R \times R$ be the subring $R^* = \{(r, r) : r \in R\} \subset R \times R$. Prove the map $\phi : R \to R^*$ which takes $\phi(r) = (r, r)$ is an isomorphism.

Problem 2.

(a) [5pts.] Let R be a commutative ring, and let $\phi : R[x] \to R$ be the map

$$\phi(a_n x^n + \cdots + a_1 x + a_0) = a_0$$

that takes each polynomial to its constant term. Prove that ϕ is a surjective homomorphism of rings.

(b) [5pts.] Let R be a commutative ring, and let $f(x) \in R[x]$. Give an example to show that the polynomial function

$$\begin{aligned} f : R &\to R \\ a &\mapsto f(a) \end{aligned}$$

is not necessarily a homomorphism.

Problem 3.

(a) [5pts.] State the Rational Root Theorem.

(b) [5pts.] Decide whether the following polynomials are irreducible in $\mathbb{Q}[x]$, justifying your answers.

- $f(x) = x^4 + 3x + 1$
- $g(x) = x^{372} + 3x^{87} - 9x^{19} + 15x^{13} + 21$
- $h(x) = x^4 + 7x^2 + 1$

Problem 4.

Let F be a field.

(a) [4pts.] Define the greatest common divisor $f(x)$ and $g(x)$ of two polynomials in $F[x]$.

(b) [3pts.] What is the greatest common divisor of $x^3 + 1$ and $x^4 + x^3 + x^2$ in $\mathbb{Z}_2[x]$?

(c) [3pts.] What is the greatest common divisor of $x^3 + 1$ and $x^4 + x^3 + x^2$ in $\mathbb{Q}[x]$?

Problem 5.

Consider the polynomial $p(x) = x^2 + 1$ in $\mathbb{Z}_3[x]$.

(a) [4pts.] Is $p(x)$ irreducible in $\mathbb{Z}_3[x]$?

(b) [4pts.] Write out the multiplication table for $\mathbb{Z}_3[x]/(x^2 + 1)$.

(c) [2pts.] Prove that the ring you constructed is not isomorphic to $\mathbb{Z}_9$.

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on this page unless you indicate *very clearly* that this is the case on the page of the corresponding problem.