MTH 310, Section 001
Abstract Algebra I and Number Theory

Midterm 1

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

(a) [5pts.] State the Division Algorithm.

(b) [5pts.] Prove that the cube a^3 of an integer a must be of one the forms $9k$, $9k + 1$, and $9k + 8$. [Hint: Start by applying the Division Algorithm with $q = 3$.]

Problem 2.

    (a) [5pts.] State the Fundamental Theorem of Arithmetic.

    (b) [5pts.] Let p be a prime integer. If $p|a^n$ for some a, does it follow that $p^n|a^n$?

Problem 3.

   (a) [5pts.] Suppose R is a ring with identity. What does it mean to say that an element a of R is a unit?

   (b) [5pts.] Let R be a four-element ring with identity $\{0, 1, a, b\}$ such that a and b are both units. Write down the full multiplication table for R. [Hint: What is ab?]

Problem 4.

    (a) [5pts.] Define a subring of a ring R.

    (b) [5pts.] Is $S = \{(a, b) : a + b = 0\}$ a subring of $\mathbb{Z} \times \mathbb{Z}$?

Problem 5.

Consider the ring $\mathbb{Z}_{12}$.

(a) [5pts.] What are the units and zero divisors in $\mathbb{Z}_{12}$?

(b) [5pts.] What are the solutions to the equation $x^2 + 2x = 0$ in $\mathbb{Z}_{12}$?

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on this page unless you indicate *very clearly* that this is the case on the page of the corresponding problem.