MTH 254H
Honors Multivariable Calculus

Sample Midterm 2

Instructions: You have 80 minutes to complete the exam. There are six questions, worth a total of sixty points. You may not use any books or notes. Partial credit will be given for progress toward correct solutions.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

Problem 1.

Consider the vector field $\mathbf{F}(x, y) = (x^2 + y^2, -2xy)$.

(a) [5pts.] Compute curl($\mathbf{F}$) and div($\mathbf{F}$).

(b) [5pts.] Is $\mathbf{F}$ the gradient vector field of any scalar-valued differentiable function $f(x, y)$? Justify your answer carefully.

Problem 2.

(a) [5pts.] The integral

$$\int_0^3 \int_{y^2}^9 y\cos(x^2)dxdy$$

(b) [5pts.] The integral of $f(x,y,z) = y$ over the region bounded by $y = 2x^2$, $y = 1+x^2$, $z = 0$, and $z = y+4$.

Problem 3.

Consider the curve $\mathbf{c}(t) = (\sin t, \cos t, e^t)$.

(a) [5pts.] Set up, but do not evaluate, and integral that gives the arclength of $\mathbf{c}(t)$ as $0 \leq t \leq 1$.

(b) [5pts.] Find a vector field $\mathbf{F}(x, y, z)$ for which $\mathbf{c}(t)$ is a flowline.

Problem 4.

Consider the function $f(x, y, z) = yz + xy$.

(a) [5pts.] Find the critical points of f on $\mathbb{R}^2$.

(b) [5pts.] What are the absolute maximum and minimum values of f on the intersection of the surfaces $xy = 1$ and $y^2 + z^2 = 1$?

Problem 5.

(a) [5pts.] Using any method you like, find the volume of the largest rectangular box in the first octant with three faces on the coordinate planes and one vertex on the plane $x + 2y + 3z = 6$.

(b) [5pts.] Find the absolute maximum and minimum values of $f(x, y) = xy^2$ on the region $D = \{(x, y) : x^2 + y^2 \leq 3, x \geq 0, y \geq 0\}$.

Problem 6.

(a) [5pts.] Find all of the points on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ where the tangent plane is parallel to $3x - y + 3z = 1$.

(b) [5pts.] Consider the function $f(x, y) = x^2 + y^2 - 2x - 4y$. Starting from $(-1, 3)$, in what direction should one walk to increase f as quickly as possible?

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on it unless you indicate *very clearly* on the page corresponding to the original problem that this is the case.