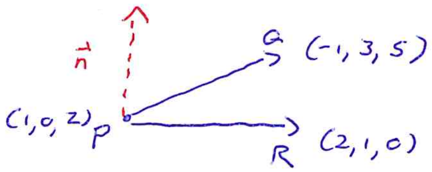
Sample Midterm 1 Solutions

1

(a)



$\vec{PQ} = (-2, 3, 3)$

$\vec{PR} = (1, 1, -2)$

$$\vec{n} = \vec{PR} \times \vec{PQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ -2 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -2 \\ 3 & 3 \end{vmatrix}\vec{i} - \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix}\vec{j} + \begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix}\vec{k}$$

$$= 9\vec{i} + \vec{j} + 5\vec{k}$$

So our plane is $9x + y + 5z = D$ for some D. But P is on the plane, so $9(1) + 0 + 5(2) = D$, hence $D = 19$ and the plane is $\boxed{9x + y + 5z = 19}$.

(b) The line is orthogonal to the plane, so its direction vector can be taken to be a normal vector $\vec{n}$ to the plane.

$$\ell(\vec{t}) = (9, 0, 9) + t(9, 1, 5) = (9 + 9t, t, 9 + 5t)$$

This is on the plane when

$$9(9 + 9t) + (1)(t) + 5(9 + 5t) = 19$$

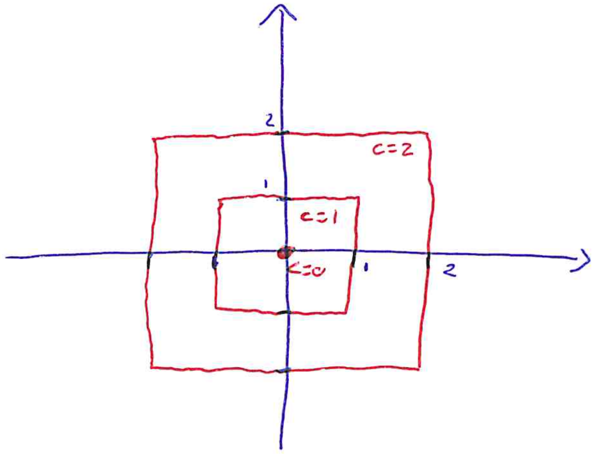
$$81 + 81t + t + 45 + 25t = 19$$

$$107t = 19 - 126$$

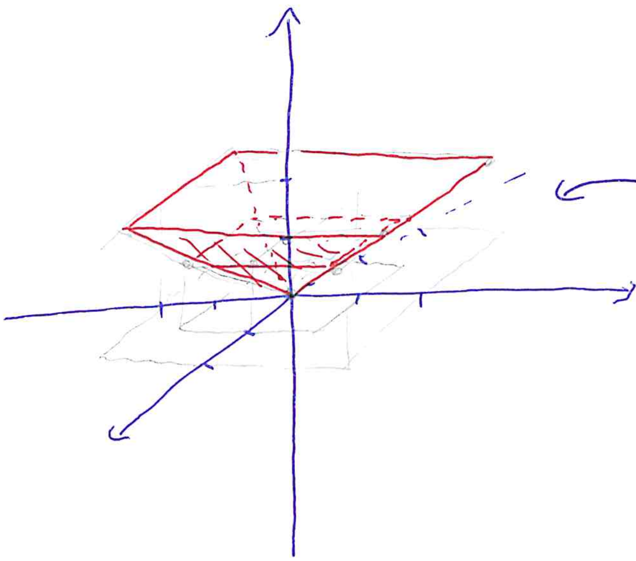
$$107t = -107 \rightarrow t = -1$$

$\boxed{\text{So at } (0, -1, 4)}$

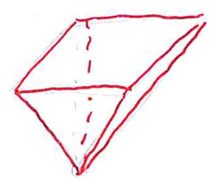
(2) (a)



(b)

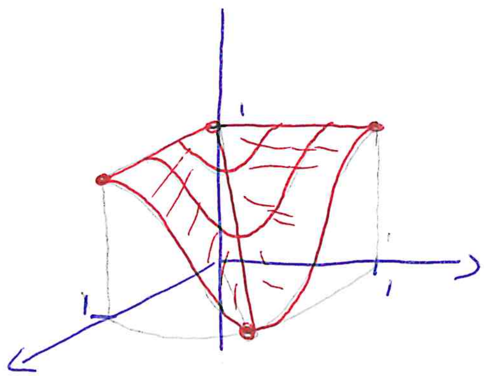


This shape is a square pyramid upside down.

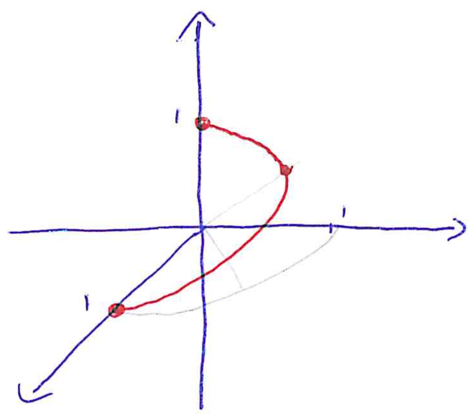


(c) $F(x,y)$ seems to be continuous everywhere and is probably not differentiable on the lines $y=x$ and $y=-x$.

3 a



b



④ (a) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x^2+y^2+z^2}{x+1}$

Notice that $\displaystyle\lim_{(x,y,z)\to(0,0,0)} x^2+y^2+z^2 = \lim_{(x,y,z)\to(0,0,0)} x^2 + \lim_{(x,y,z)\to(0,0,0)} y^2 + \lim_{(x,y,z)\to(0,0,0)} z^2$

$$= \lim_{x\to 0} x^2 + \lim_{y\to 0} y^2 + \lim_{z\to 0} z^2$$

$$= 0+0+0$$

$$= 0$$

and $\displaystyle\lim_{(x,y,z)\to(0,0,0)} x+1 = \lim_{x\to 0} x+1 = 1$. Since the limit in the denominator is nonzero, we have $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{x^2+y^2+z^2}{x+1} = \frac{0}{1} = 0$.

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$.

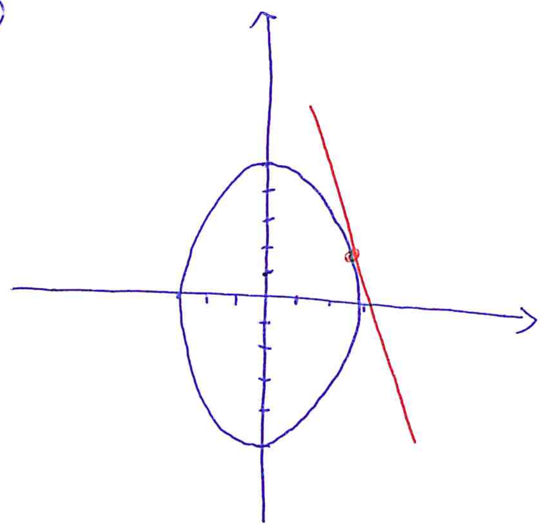
<u>Approach along $x=0$</u> $\displaystyle\frac{xy^2}{x^2+y^4} = \frac{0}{y^4} = 0 \quad \lim_{y\to 0} \frac{0}{y^4} = 0.$

<u>Approach along $x=y^2$</u> $\displaystyle\frac{y^4}{y^4+y^4} = \frac{1}{2} \quad \lim_{y\to 0} \frac{1}{2} = \frac{1}{2}$

The limit is different along two different paths to the origin, so the overall limit does not exist.

(5) (a) $\vec{c}(t) = (3\cos t, 5\sin t)$ for any interval of length at least 2π,

(b)



$\vec{c}'(t) = (-3\sin t, 5\cos t)$

$\vec{c}\left(\frac{\pi}{6}\right) = \left(\frac{3\sqrt{3}}{2}, \frac{5}{2}\right)$

$\vec{c}'\left(\frac{\pi}{6}\right) = \left(\frac{-3}{2}, \frac{5\sqrt{3}}{2}\right)$

$\vec{l}(t) = \left(\frac{3\sqrt{3}}{2}, \frac{5}{2}\right) + \left(t - \frac{\pi}{6}\right)\left(\frac{-3}{2}, \frac{5\sqrt{3}}{2}\right)$

(6) $F(x,y) = e^{2x+3y}$

(a) $\begin{bmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \end{bmatrix} = \begin{bmatrix} 2e^{2x+3y} & 3e^{2x+3y} \end{bmatrix}$

(b) $z = F(x_0, y_0) + \frac{\partial F}{\partial x}\Big|_{(x_0,y_0)} (x - x_0) + \frac{\partial F}{\partial y}\Big|_{(x_0,y_0)} (y - y_0)$

$z = 1 + 2e^{2(0)+3(0)}(x-0) + 3e^{2(0)+3(0)}(y-0)$

$\boxed{z = 1 + 2x + 3y}$