

MTH 254H
Honors Multivariable Calculus

Sample Midterm 1

Instructions: You have 80 minutes to complete the exam. There are six questions, worth a total of sixty points. You may not use any books or notes. Partial credit will be given for progress toward correct solutions.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: _____

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

Problem 1.

- (a) [5pts.] Find an equation for the plane in \mathbb{R}^3 that passes through the points $(1, 0, 2)$, $(-1, 3, 5)$, and $(2, 1, 0)$.
- (b) [5pts.] Parametrize the line that is perpendicular to the plane you found in part (a) and passes through the point $(9, 0, 9)$. Where does this line intersect the plane?

Problem 2.

Let $f(x, y) = \max\{|x|, |y|\}$. (That is, the value of the function is the larger of the values of $|x|$ and $|y|$ at (x, y)).

- (a) [4pts.] Sketch level curves for $f(x, y)$ with $c = 0, 1, 2$.
- (b) [4pts.] Sketch a graph the surface $z = f(x, y)$.
- (c) [2pts.] Where do you think $f(x, y)$ is continuous? Where do you think $f(x, y)$ is differentiable? (Your answer to this part is not expected to be rigorously justified.)

Problem 3.

Sketch the following.

- (a) [5pts.] The surface given by $z = 1 - r \sin(2\theta)$, $0 \leq r \leq 1$, $0 \leq \theta \leq \frac{\pi}{2}$, in cylindrical coordinates.
- (b) [5pts.] The curve given by $\rho = 1$, $\cos \phi = \sin \theta$, $0 \leq \theta \leq \frac{\pi}{2}$ in spherical coordinates.

Problem 4.

For each of the following, either compute the limit or prove it does not exist.

(a) [5pts.]

$$\lim_{(x,y,z) \rightarrow (0,0)} \frac{x^2 + y^2 + z^2}{x + 1}$$

(b) [5pts.]

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + y^4}$$

Problem 5.

Let C be the ellipse given by $\frac{x^2}{9} + \frac{y^2}{25} = 1$.

(a) [5pts.] Give a parametrization of C .

(b) [5pts.] Give an equation for the tangent line to C at $(\frac{3\sqrt{3}}{2}, \frac{5}{2})$.

Problem 6.

Let $f(x, y) = e^{2x+3y}$.

- (a) [5pts.] Compute the matrix of partial derivatives of f .
- (b) [5pts.] What is the equation of the tangent plane to $z = e^{2x+3y}$ at $(0, 0, 1)$?

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on it unless you indicate *very clearly* on the page corresponding to the original problem that this is the case.