MTH 254H
Honors Multivariable Calculus

Sample Final

Instructions: You have two hours to complete the exam. There are eight questions, worth a total of eighty points. You may not use any books or notes. Partial credit will be given for progress toward correct solutions.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total:	80	

Problem 1.

(a) [5pts.] What is the area enclosed by the loop of the curve $\mathbf{c}(t) = (t^2 - 1, t^3 - t)$ as $-1 \leq t \leq 1$?

(b) [5pts.] What is the work done by the vector field $\mathbf{F}(x, y, z) = (2xyz + \sin x, x^2 z, x^2 y)$ on a particle moving along the curve $\mathbf{c}(t) = (e^{t^2}, 5t + 1, 6t^3 - 7)$ on a particle moving along the curve $\mathbf{c}$ from time $t = 0$ to $t = 1$?

Problem 2.

Let $\mathbf{F}(x, y, z) = (z^2 x, \frac{1}{3}y^3 + \tan z, x^2 z + y^2)$.

(a) [5pts.] What is the flux of $\mathbf{F}$ across the unit disk $\{(x, y, 0) : x^2 + y^2 \leq 1\}$, oriented downward?

(b) [5pts.] What is the flux of $\mathbf{F}$ across the upper half of the unit sphere, oriented upward? [Hint: If you are trying to do this directly, you are working too hard.]

Problem 3.

Consider the integral

$$\int\int_B (x+y)dA$$

where B is the parallelogram with corners at $(0,1)$, $(1,0)$, $(3,4)$ and $(4,3)$.

(a) [5pts.] Find a linear transformation $T(u,v) = (x(u,v), y(u,v))$ which maps a rectangle in the plane to B.

(b) [5pts.] Compute the integral above.

Problem 4.

Consider the functions $f(x,y) = (x^2 + 2y, xy^3 - 1)$ and $g(x,y) = (\ln x, e^y, x+y)$.

(a) [5pts.] Find $\mathbf{D}f(x,y)$ and $\mathbf{D}g(x,y)$.

(b) [5pts.] What is $\mathbf{D}(g \circ f)(1,0)$?

Problem 5.

Consider the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 1$ and $z = 2$. Suppose that it has density $\delta(x, y, z) = (x^2 + y^2)z^2$.

(a) [5pts.] What is the mass of the solid?

(b) [5pts.] Where is the center of mass of the solid?

Problem 6.

Consider the function $f(x, y, z) = xe^z + y \cos x$.

(a) [5pts.] Suppose you are standing at $(0, 1, 5)$. In what direction should you move so that f increases the fastest?

(b) [5pts.] What is the tangent plane to $f(x, y, z) = 2$ at $(0, 2, 7)$?

Problem 7.

Either compute the limit, or prove it does not exist.

(a) [5pts.]

$$\lim_{(x,y)\to(0,0)} \frac{(x-y)^2}{x^2+y^2}$$

(b) [5pts.]

$$\lim_{(x,y)\to(0,0)} \frac{e^{xy}}{x+1}$$

Problem 8.

Consider the function $f(x, y) = y + x \sin y$.

(a) [3pts.] Find all the critical points of f.

(b) [4pts.] For each critical point you found in part (a), determine whether it a local maximum, a local minimum, or neither.

(c) [3pts.] What is the absolute maximum of f on the rectangle $[0, 1] \times [0, \frac{\pi}{2}]$?

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on it unless you indicate *very clearly* on the page corresponding to the original problem that this is the case.

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