Problem 1. *5pts.*

Let $f(x, y) = (\sin x + \cos y, xy^2)$, and let $\mathbf{c}(t)$ be a path in the plane such that $\mathbf{c}(0) = 0$ and $\mathbf{c}'(0) = (1, 3)$. Consider that path $\mathbf{p}(t) = f(\mathbf{c}(t))$. What is $\mathbf{p}'(0)$?

$\vec{p}\,'(0) = \vec{D}f(\vec{c}(0)) \cdot \vec{c}\,'(0)$ by the Chain Rule.

$$\vec{D}f(x,y) = \begin{pmatrix} \cos x & -\sin y \\ y^2 & 2xy \end{pmatrix}$$

$$\vec{D}f(\vec{c}(0)) = \vec{D}f(0,0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\vec{p}\,'(0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Problem 2. *5pts.*

What are the largest and smallest values taken by the function $f(x, y, z) = x + yz$ on region $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1\}$?

<u>Interior Critical Points</u>

$$\frac{\partial F}{\partial x} = 1 \qquad \frac{\partial F}{\partial y} = z \qquad \frac{\partial F}{\partial z} = y$$

$\Big\}$ Cannot all be zero $\leadsto$ no interior extrema

<u>Boundary Extrema</u> We use the method of Lagrange multipliers, optimizing $f(x,y,z) = x+yz$ ~~an~~ with respect to the constraint $1 = g(x,y,z) = x^2+y^2+z^2$.

- $\nabla F = (1, z, y)$ $\qquad \nabla g = (2x, 2y, 2z)$

  $\leftarrow$ Not zero anywhere on the unit sphere.

- $\nabla F = \lambda \nabla g$ For some $\lambda \in \mathbb{R}$

$$\begin{cases} 1 = 2\lambda x \\ z = 2\lambda y \\ y = 2\lambda z \\ x^2+y^2+z^2 = 1 \end{cases}$$

- We see

  $z = 2\lambda y = 2\lambda(2\lambda z) = 4\lambda^2 z$

  So $z = 4\lambda^2 z \Rightarrow z = 0$ or $\lambda = \pm\frac{1}{2}$.

- If $z = 0$, $y = 2\lambda(0) = 0$, so

  $(x,y,z) = (\pm 1, 0, 0)$.

- If $\lambda = \frac{1}{2}$, $x = \frac{1}{2\lambda} = 1$, so $(x,y,z) = (1,0,0)$

- If $\lambda = -\frac{1}{2}$, $x = -1$, so $(x,y,z) = (-1,0,0)$

$f(1,0,0) = 1+0 = 1$ $\}$ Global max

$F(-1,0,0) = -1+0 = -1$ $\}$ Global min