MTH 254H
Honors Multivariable Calculus

Midterm 2

Instructions: You have 80 minutes to complete the exam. There are six questions, worth a total of sixty points. You may not use any books or notes. Partial credit will be given for progress toward correct solutions.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

Problem 1.

You are on a mountain whose height of the plane is given by $h(x, y) = xy^2 - y + \cos(\pi x)$ m. Suppose you are standing above the point $(1, 1)$.

(a) [3pts.] At what rate will your height above the plane increase if you walk due north at speed 1 m/s?

(b) [4pts.] In what direction should you walk to ascend the mountain as quickly as possible?

(c) [3pts.] In what directions can you walk to stay at your current height?

Problem 2.

Let $\mathbf{F}(x, y, z) = (2xy + z\cos y, x^2 - xz\sin y, x\cos y)$.

(a) [5pts.] Find $\mathrm{curl}(\mathbf{F})$ and $\mathrm{div}(\mathbf{F})$.

(b) [5pts.] Does there exist a scalar-valued function such that $f(x, y, z)$ such that $\mathbf{F}(x, y, z) = \nabla f$? Does there exist a vector field $\mathbf{G}(x, y, z)$ such that $\mathbf{F} = \mathrm{curl}(\mathbf{G})$?

Problem 3.

(a) [5pts.] Find the maximum and minimum values of $f(x, y, z) = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ on the surface $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 1$.

(b) [5pts.] Using any method you like, find the points on the surface $xy^2z^3 = 2$ that are closest to the origin.

Problem 4.

Consider the function $f(x, y) = 4xy^2 - x^2y^2 - xy^3$.

(a) [5pts.] Find all of the critical points on f on $\mathbb{R}^2$. For each critical point you find with $y \neq 0$, decide whether f has a local maximum, a local mimimum, or no local extremum at that point.

(b) [5pts.] Find the absolute maximum and minimum of f on the closed triangular region with vertices $(0, 0)$, $(0, 6)$, and $(6, 0)$.

Problem 5.

Compute the following.

(a) [5pts.] The volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x+y = 4$, and $x = z - y - 1$.

(b) [5pts.] The integral

$$\int_0^1 \int_x^{\sqrt{x}} e^{\frac{x}{y}} \, dy dx$$

Problem 6.

Consider the curve $\mathbf{c}(t) = (\frac{1}{2}t^2, \sqrt{2}t, \ln t)$.

(a) [5pts.] Verify that $\mathbf{c}(t)$ is a flowline of the vector field $\mathbf{F}(x, y, z) = (\frac{\sqrt{2}}{y}, \sqrt{2}, \frac{y}{\sqrt{2}})$.

(b) [5pts.] What is the length of $\mathbf{c}(t)$ over $1 \leq t \leq 2$?

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on it unless you indicate *very clearly* on the page corresponding to the original problem that this is the case.

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