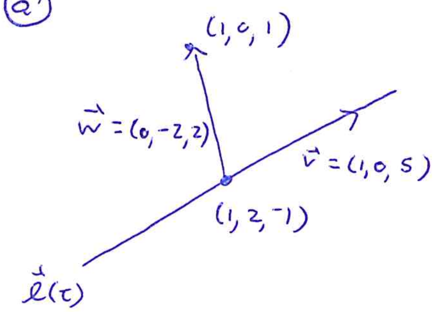
Midterm 1 Solutions

1 (a)



The plane contains $\vec{v}=(1,0,5)$ and $\vec{w}=(0,-2,2)$, so a potential normal is

$$\vec{n}=\vec{v}\times\vec{w}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 0 & 5\\ 0 & -2 & 2\end{vmatrix}=(10,-2,-2).$$

So our plane is $10x-2y-2z=D$ for some D. But $(1,0,1)$ is on the plane, so $D=10(1)-2(0)-2(1)=8$. Hence

$\boxed{10x-2y-2z=8}$ is an equation for the plane.

(b) $\vec{n}_1=(10,-2,-2)$

$\vec{n}_2=(2,3,-1)$

The angle between these vectors is

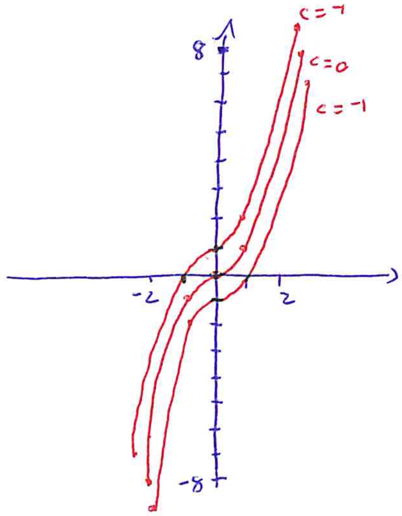
$$\theta=\cos^{-1}\left(\frac{\vec{n}_1\cdot\vec{n}_2}{\|\vec{n}_1\|\,\|\vec{n}_2\|}\right)$$

$$=\cos^{-1}\left(\frac{20-6+2}{\sqrt{108}\,\sqrt{14}}\right)$$

$$=\cos^{-1}\left(\frac{16}{6\sqrt{3}\,\sqrt{14}}\right)$$

$$=\cos^{-1}\left(\frac{8}{3\sqrt{42}}\right)$$

(2) (a)

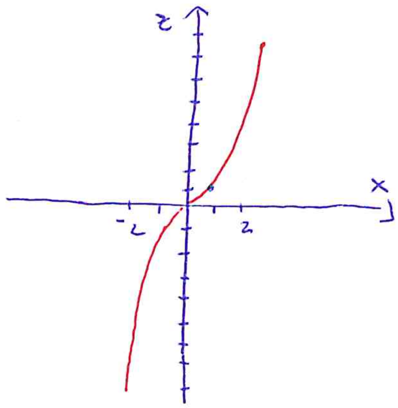


$c = x^3 - y$

$y = x^3 - c$

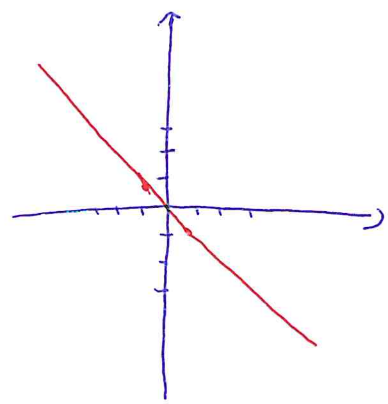
(b) xz-plane $y = 0$

$z = x^3$



yz-plane $x = 0$

$z = -y$



3 (a) $F(x,y) = (x\sin y,\ y^2 + 2xy)$

$$\underset{F_1(x,y)}{\uparrow} \qquad \underset{F_2(x,y)}{\uparrow}$$

$$\begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{bmatrix} = \begin{bmatrix} \sin y & x\cos y \\ 2y & 2y + 2x \end{bmatrix}$$

Yes, the partials are all clearly continuous functions of x and y, so F is differentiable everywhere (i.e., the linear map determined by this matrix is always a good local approximation to the function).

(b) $F(x,y) = xe^y + 2$ $\qquad \frac{\partial F}{\partial x} = e^y \qquad \frac{\partial F}{\partial y} = xe^y$

Tangent Plane at $(3,0,5)$

$$z = F(x_0,y_0) + \left.\frac{\partial F}{\partial x}\right|_{(x_0,y_0)} (x - x_0) + \left.\frac{\partial F}{\partial y}\right|_{(x_0,y_0)} (y - y_0)$$

$$z = 5 + (e^0)(x-3) + 3e^0(y-0)$$

$$z = 5 + (x-3) + 3y$$

$$\boxed{z = 2 + x + 3y}$$

4 (a) $$\lim_{(x,y)\to(0,0)} \frac{(x+y)^2}{x^2+y^2}$$

Approach along $x=0$

$$\frac{(x+y)^2}{x^2+y^2} = \frac{y^2}{y^2} = 1$$

$$\lim_{y\to 0} 1 = 1$$

Approach along $x=-y$

$$\frac{(x+y)^2}{x^2+y^2} = \frac{0^2}{2y^2} = 0$$

$$\lim_{y\to 0} 0 = 0$$

The limit does not exist.

(b) $$\lim_{(x,y)\to(0,1)} \frac{xy}{x^2-y^2}$$

Note that $\lim_{(x,y)\to(0,1)} xy = 0(1) = 0$ and $\lim_{(x,y)\to(0,1)} x^2-y^2 = 0-1 = -1 \neq 0$

So $$\lim_{(x,y)\to(0,1)} \frac{xy}{x^2-y^2} = \frac{0}{-1} = 0.$$

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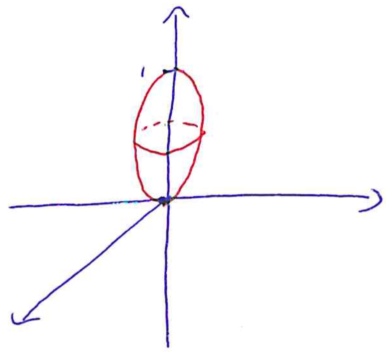
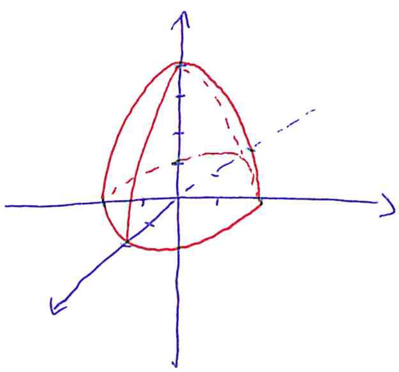
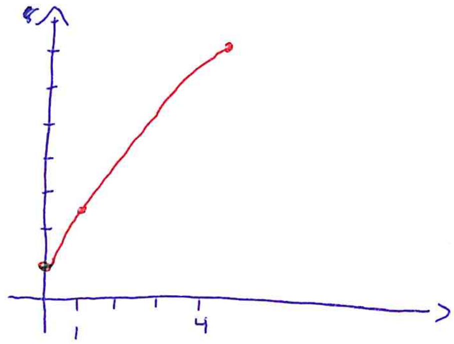
(a)



(b)



⑥ (a) $\vec{c}(t) = (t^2, e^t)$



(b) At $(1, e)$, we have $t = 1$.

$\vec{c}\,'(t) = (2t, e^t)$

$\vec{c}\,'(1) = (2, e)$

$\vec{l}(t) = (1, e) + (t-1)(2, e) = (1 + 2t, et)$