

**MTH 254H**  
**Honors Multivariable Calculus**

**Midterm 1**

**Instructions:** You have 80 minutes to complete the exam. There are six questions, worth a total of sixty points. You may not use any books or notes. Partial credit will be given for progress toward correct solutions.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: \_\_\_\_\_

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total:	60	

**Problem 1.**

- (a) [5pts.] Find an equation for the plane which contains the point  $(1, 0, 1)$  and the line  $\ell(t) = (1, 2, -1) + t(1, 0, 5)$ .
- (b) [5pts.] The angle at which two (nonparallel) planes intersect is the angle between their normal vectors. With this in mind, at what angle do the plane you found in part (a) and the plane  $2x + 3y - z = 7$  intersect?

**Problem 2.**

Let  $f(x, y) = x^3 - y$ .

- (a) [5pts.] Draw the level curves of  $f$  for  $c = -1, 0, 1$ .
- (b) [5pts.] Draw the sections of the graph of  $f(x, y)$  given by intersecting the graph with the  $xz$ -plane and with the  $yz$ -plane.

**Problem 3.**

- (a) [5pts.] Compute the matrix of partial derivatives of the function  $f(x, y) = (x \sin y, y^2 + 2xy)$ . Is  $f$  differentiable? Justify your answer.
- (b) [5pts.] What is the equation of the tangent plane to the surface  $z = xe^y + 2$  at the point  $(3, 0, 5)$ ?

**Problem 4.**

For each of the following, either compute the limit or prove it does not exist.

(a) [5pts.]

$$\lim_{(x,y) \rightarrow (0,0)} \frac{(x+y)^2}{x^2 + y^2}$$

(b) [5pts.]

$$\lim_{(x,y) \rightarrow (0,1)} \frac{xy}{x^2 - y^2}$$

**Problem 5.**

Sketch the following.

- (a) [5pts.] The surface given by the equation  $\rho = \cos \phi$ , for  $\phi \in [0, \frac{\pi}{2}]$  in spherical coordinates.
- (b) [5pts.] The surface given by the equation  $z = 4 - r^2$ ,  $0 \leq r \leq 2$ , in cylindrical coordinates.

**Problem 6.**

- (a) [5pts.] Sketch the curve parametrized by  $\mathbf{c}(t) = (t^2, e^t)$  on the interval  $t \in [0, 2]$ .
- (b) [5pts.] Give an equation for the tangent line to the curve at  $(1, e)$ .

This page is for scratch work. Feel free to tear it off. Do not write anything you want graded on it unless you indicate *very clearly* on the page corresponding to the original problem that this is the case.