MTH 254H: Homework 3

Due: September 22, 2017

1. Read Sections 1.5, 2.1-2 in Marsden and Tromba.

2. Do problems 1.3.6, 1.3.8, 1.3.16, 1.3.19, 1.3.22, 1.3.46, 1.4.3, 1.4.5, 1.4.6, 1.4.7, 1.4.10, 1.5.5, 1.5.16 in Marsden and Tromba.

3. Given vectors $\mathbf{v_1}, \cdots \mathbf{v_n}$, a linear combination of these vectors is a sum of the form $\lambda_1 \mathbf{v_1} + \cdots + \lambda_n \mathbf{v_n}$. The set of vectors $\{\mathbf{v_1}, \cdots, \mathbf{v_n}\}$ is said to be *linearly dependent* if there is some linear combination of the vectors such that $\lambda_1 \mathbf{v_1} + \cdots \lambda_n \mathbf{v_n} = 0$, and *linearly independent* otherwise.

   - Is the set $\{(1,2),(2,4)\}$ linearly independent in $\mathbb{R}^2$? How about the set $\{(1,7),(2,0)\}$?
   - Is the set $\{(2,0,1)\}$ linearly independent in $\mathbb{R}^3$? How about $\{(1,0,4),(7,2,1),(0,2,0)\}$?
   - Show that the set of two vectors $\{\mathbf{a},\mathbf{b}\}$ in $\mathbb{R}^2$ is linearly dependent if and only if the 2×2 square matrix with rows $\{\mathbf{a},\mathbf{b}\}$ has determinant zero. (Hint: This is quite straightforward if you write it out in coordinates.)
   - Using the properties of the determinant proved in class, show that if the set of three vectors $\{\mathbf{a},\mathbf{b},\mathbf{c}\}$ in $\mathbb{R}^3$ is linearly dependent, then the 3×3 square matrix with rows $\{\mathbf{a},\mathbf{b},\mathbf{c}\}$ has determinant zero. (The converse is also true but requires either more linear algebra technology or quite a lot of algebra to prove.)

4. A linear transformation $f\colon \mathbb{R}^n \to \mathbb{R}^n$ is said to be an *isometry* if for every $\mathbf{a} \in \mathbb{R}^n$, $f(\mathbf{a}) \cdot f(\mathbf{b}) = \mathbf{a} \cdot \mathbf{b}$. This implies, in particular, that the length of $f(\mathbf{a})$ is equal to the norm of $\mathbf{a}$ and for every $\mathbf{a},\mathbf{b} \in \mathbb{R}^n$, the angle between $f(\mathbf{a})$ and $f(\mathbf{b})$ is the same as the angle between $\mathbf{a}$ and $\mathbf{b}$.

   - Show that the transformations R_θ and F from Question 4 of Homework 2 are isometries.
   - Suppose $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ is an isometry. What possible forms can its matrix with respect to the standard basis take? (Hint: Let $f(\mathbf{e_1}) = (c_1, d_1)$ and $f(\mathbf{e_2}) = (c_2, d_2)$, and work out what restrictions you can place on the numbers c_1, d_1, c_2, and d_2.)

Notes on homework prep:

- There will be some discussion of Questions 3 and 4 in section.