MTH 254H: Homework 2

Due: September 15, 2017

1. Remember that the first twenty-minute quiz is Thursday, September 14 at the beginning of discussion section. It will cover the material through lecture on Friday, September 8; that is, Sections 1.1-2 and Section 1.3 up to the definition of the cross product.

2. Read Sections 1.3-4 in Marsden and Tromba.

3. Do problems 1.2.13, 1.2.16, 1.2.19, 1.2.25, 1.2.26, 1.2.29, 1.2.31, 1.2.34, 1.3.7, 1.3.9, 1.3.10, 1.3.13, 1.3.15 in Marsden and Tromba.

4. A map \( f : \mathbb{R}^n \to \mathbb{R}^m \) is called a \textit{linear transformation} if it preserves addition and scalar multiplication; that is, if given \( \mathbf{a}, \mathbf{b} \) vectors in \( \mathbb{R}^n \) and \( \alpha \in \mathbb{R} \), we have that \( f(\mathbf{a} + \mathbf{b}) = f(\mathbf{a}) + f(\mathbf{b}) \) and \( f(\alpha \mathbf{a}) = \alpha f(\mathbf{a}) \).

   • Consider a line \( \ell(t) = \mathbf{a} + t\mathbf{v} \) in \( \mathbb{R}^3 \) as defined in class. This is a map \( \ell : \mathbb{R} \to \mathbb{R}^3 \). Under what circumstances is \( \ell \) a linear transformation?

   • Suppose \( f : \mathbb{R}^2 \to \mathbb{R}^2 \) is a linear transformation. Show that given \( \mathbf{a} \in \mathbb{R}^2 \), \( f(\mathbf{a}) \) is determined by \( f(\mathbf{i}) \) and \( f(\mathbf{j}) \). [In other words, if you know that \( f((1,0)) = (c_1,d_1) \) and that \( f((0,1)) = (c_2,d_2) \), show that you can give a formula for \( f((a_1,a_2)) \) in terms of the numbers \( a_1, a_2, c_1, c_2, d_1, d_2 \).] Under these circumstances, the matrix

     \[
     \begin{pmatrix}
     c_1 & c_2 \\
     d_1 & d_2
     \end{pmatrix}
     \]

     is called the matrix of the linear transformation with respect to the standard basis. Use this to express the formula you found above in terms of matrix multiplication.

   • Show that rotation \( R_\theta \) counterclockwise by an angle \( \theta \) in the plane is a linear transformation, and compute its matrix with respect to the standard basis. Do the same for reflection \( F \) across the \( x \)-axis.

Notes on homework prep:

• For problems 1.2.29, 1.2.31, and 1.2.34, you will first want to read the subsection entitled "Physical Applications of Vectors" in Section 1.2.

• There will be some discussion of Question 4 in section.