MTH 254H: Homework 2

Due: September 15, 2017

1. Remember that the first twenty-minute quiz is Thursday, September 14 at the *beginning* of discussion section. It will cover the material through lecture on Friday, September 8; that is, Sections 1.1-2 and Section 1.3 up to the definition of the cross product.

2. Read Sections 1.3-4 in Marsden and Tromba.

3. Do problems 1.2.13, 1.2.16, 1.2.19, 1.2.25, 1.2.26, 1.2.29, 1.2.31, 1.2.34, 1.3.7, 1.3.9, 1.3.10, 1.3.13, 1.3.15 in Marsden and Tromba.

4. A map $f\colon \mathbb{R}^n \to \mathbb{R}^m$ is called a *linear transformation* if it preserves addition and scalar multiplication; that is, if given $\mathbf{a}, \mathbf{b}$ vectors in $\mathbb{R}^n$ and $\alpha \in \mathbb{R}$, we have that $f(\mathbf{a}+\mathbf{b}) = f(\mathbf{a}) + f(\mathbf{b})$ and $f(\alpha\mathbf{a}) = \alpha f(\mathbf{a})$.

   - Consider a line $\ell(t) = \mathbf{a} + t\mathbf{v}$ in $\mathbb{R}^3$ as defined in class. This is a map $\ell\colon \mathbb{R} \to \mathbb{R}^3$. Under what circumstances is ℓ a linear transformation?
   - Suppose $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation. Show that given $\mathbf{a} \in \mathbb{R}^2$, $f(\mathbf{a})$ is determined by $f(\mathbf{i})$ and $f(\mathbf{j})$. [In other words, if you know that $f((1,0)) = (c_1, d_1)$ and that $f((0,1)) = (c_2, d_2)$, show that you can give a formula for $f((a_1, a_2))$ in terms of the numbers $a_1, a_2, c_1, c_2, d_1, d_2$.] Under these circumstances, the matrix

     $$\begin{pmatrix} c_1 & c_2 \\ d_1 & d_2 \end{pmatrix}$$

     is called the matrix of the linear transformation with respect to the standard basis. Use this to express the formula you found above in terms of matrix multiplication.
   - Show that rotation R_θ counterclockwise by an angle θ in the plane is a linear transformation, and compute its matrix with respect to the standard basis. Do the same for reflection F across the x-axis.

Notes on homework prep:

- For problems 1.2.29, 1.2.31, and 1.2.34, you will first want to read the subsection entitled "Physical Applications of Vectors" in Section 1.2.
- There will be some discussion of Question 4 in section.