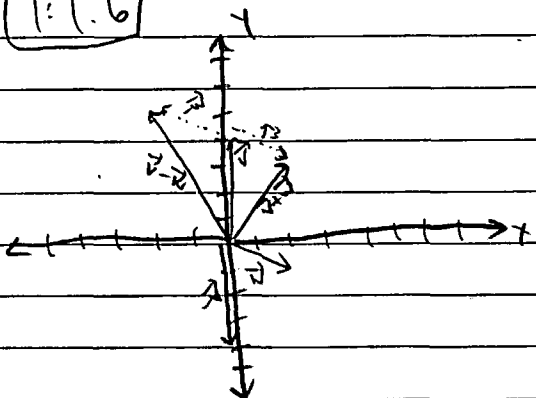


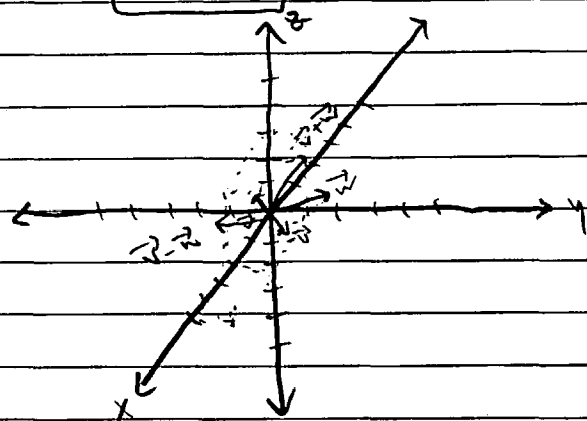
Homework #1

1/8

1.1.6



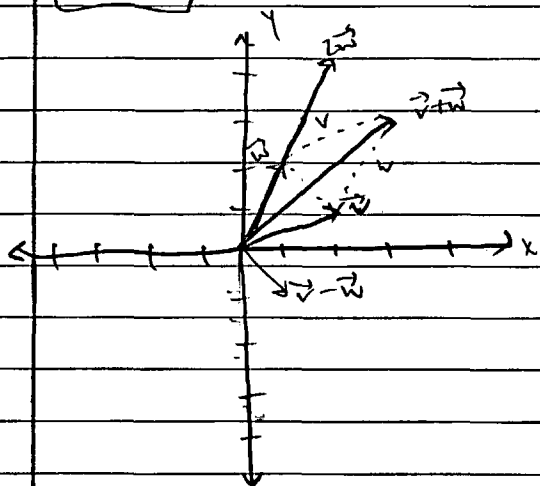
1.1.8



$$\vec{v} + \vec{w} = (0, 1, 2)$$

$$\vec{v} - \vec{w} = (4, 1, 4)$$

1.1.9



1.1.16

Passes thru: $(0, 2, 1) = \vec{a}$

In direction vector: $(2, 0, -1) = \vec{v}$

$$l(t) = \vec{a} + t\vec{v}$$

$$= (0, 2, 1) + t(2, 0, -1)$$

$$= (2t, 2, 1-t) \quad \star$$

Equivalently:

$$x(t) = 2t$$

$$y(t) = 2$$

$$z(t) = 1-t$$

$$\forall t \in \mathbb{R}$$

Homework #1

2/8

1.1.17

Passes thru $(-1, -1, -1) = \vec{s}_1$

and $(1, -1, 2) = \vec{s}_2$

$$\vec{v} = \vec{s}_2 - \vec{s}_1 = (2, 0, 3)$$

$$= (-2, 0, -3)$$

Choose either point to be \vec{s} :

$$l(t) = \vec{s} + t\vec{v} = (-1, -1, -1) + t(2, 0, 3)$$

$$= (-1+2t, -1, -1+3t) \checkmark$$

1.1.21

If they're on the same line, the line from point 1 to point 2 should pass thru point 3. That line is:

$$\vec{s} = \vec{s}_1 = (2, 3, -4)$$

$$\vec{v} = \vec{s}_2 - \vec{s}_1 = (0, -2, 3)$$

$$l(t) = (2, 3-2t, -4+3t)$$

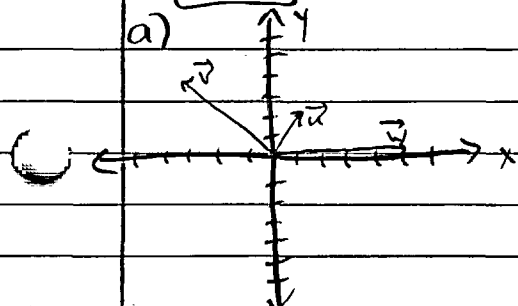
Does there exist a t , such that

$$l(t) = (2, 7, -10)? \text{ Yep - it's}$$

$t = -2$, so they're colinear. \checkmark

1.1.22

a)



$$b) \vec{w} = \lambda_1 \vec{u} + \lambda_2 \vec{v}$$

$$(5, 0) = \lambda_1(1, 2) + \lambda_2(-3, 4)$$

$$5 = \lambda_1 - 3\lambda_2$$

$$0 = 2\lambda_1 + 4\lambda_2$$

$$\Rightarrow \lambda_1 = -2\lambda_2, \lambda_2 = -1,$$

$$\lambda_1 = 2$$

$$(\lambda_1, \lambda_2) = (2, -1) \checkmark$$

1.1.27

$$l_1(t) = (3t+2, t-1, 6t+1)$$

$$l_2(s) = (3s-1, s-2, s)$$

How do we solve? Set $l_1(t) = l_2(s)$

and solve the system:

$$3t+2 = 3s-1$$

$$t-1 = s-2 \Rightarrow t = s-1$$

$$6t+1 = s$$

$$3(s-1)+2 = 3s-1$$

$$3s-1 = 3s-1 \text{ good}$$

$$6(s-1)+1 = s$$

$$6s-5 = s$$

$$s = 1 \text{ good}$$

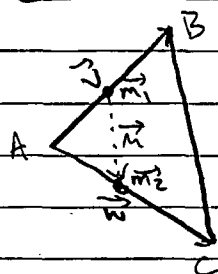
When $s=1, t=0$:

$$l_1(0) = (2, -1, 1)$$

$$l_2(1) = (2, -1, 1)$$

So they DO overlap. \checkmark

(*) 1.1.32



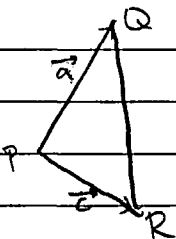
Let ABC be a triangle, and let \vec{v} denote \overrightarrow{AB} and \vec{w} denote \overrightarrow{AC} . Then per our construction, the side \overline{BC} is given by the vector $\vec{w} - \vec{v}$, and has length $|\vec{w} - \vec{v}|$.

Consider the line segment formed by the midpoints of sides AB and AC . By definition of midpoints, its points are given by the vectors $\frac{1}{2}\vec{v}$ and $\frac{1}{2}\vec{w}$ (denoted \vec{m}_1 and \vec{m}_2 in the picture) and its vector \vec{m} is the difference of its endpoints as before: $\vec{m} = \vec{m}_2 - \vec{m}_1 = \frac{1}{2}\vec{w} - \frac{1}{2}\vec{v} = \frac{1}{2}(\vec{w} - \vec{v})$.

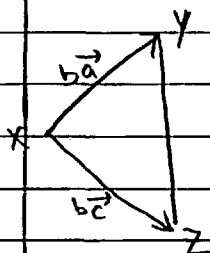
There exists a nonzero scalar α such that $\overrightarrow{BC} = \alpha(\vec{m})$:
 $(\vec{w} - \vec{v}) = \alpha(\frac{1}{2}\vec{w} - \frac{1}{2}\vec{v}) = \frac{\alpha}{2}\vec{w} - \frac{\alpha}{2}\vec{v}$ for $\alpha = 2$,
 which implies that \overline{BC} and \vec{m} are parallel. We found α to be 2, so $|\overline{BC}| = 2|\vec{m}|$, and the line joining the midpoint is half the length of the opposite side.

1.1.33

Let PQR be a triangle in space and let $b \in \mathbb{R}$, $b > 0$, and let the vectors representing \overrightarrow{PQ} and \overrightarrow{PR} be given by \vec{a} and \vec{c} respectively, and thus $\overrightarrow{QR} = \vec{c} - \vec{a}$.



Consider the figure XYZ such that \overrightarrow{XY} and \overrightarrow{XZ} are given by $b\vec{a}$ and $b\vec{c}$. Since \overrightarrow{XY} and \overrightarrow{XZ} are scalar multiples of \overrightarrow{PQ} and \overrightarrow{PR} , they are parallel to their counterparts. Per our construction, the side \overline{YZ} is given by $\overrightarrow{XZ} - \overrightarrow{XY} = b\vec{c} - b\vec{a} = b(\vec{c} - \vec{a}) = b(\overrightarrow{QR})$, so it is also parallel to its counterpart \overline{QR} in PQR , with each line segment scaled by b ; so XYZ must be a triangle with sides parallel to PQR with sides scaled by b .



Homework #1

4/81

1.1.35

Let (a, b, c) denote a compound with particles $C_a H_b O_c$. Then

$$CO = C_1 H_0 O_1 = (1, 0, 1)$$

$$H_2O = C_0 H_2 O_1 = (0, 2, 1)$$

$$H_2 = C_0 H_2 O_0 = (0, 2, 0) \quad \text{and}$$

$$CO_2 = C_1 H_0 O_2 = (1, 0, 2).$$

Then, we can represent $CO + H_2O = H_2 + CO_2$ as

$$(1, 0, 1) + (0, 2, 1) = (0, 2, 0) + (1, 0, 2)$$

$$(1, 2, 2) = (1, 2, 2) \quad \text{and the relationship holds } \checkmark$$

* 1.2.6

$$\vec{u} = 15\vec{i} - 2\vec{j} + 4\vec{k} = (15, -2, 4)$$

$$\vec{v} = \pi\vec{i} + 3\vec{j} - \vec{k} = (\pi, 3, -1)$$

$$\vec{u} \cdot \vec{u} = (15 \cdot 15) + (-2 \cdot -2) + (4 \cdot 4) = 225 + 4 + 16 = 245$$

$$\vec{v} \cdot \vec{v} = (\pi \cdot \pi) + (3 \cdot 3) + (-1 \cdot -1) = \pi^2 + 9 + 1 = \pi^2 + 10$$

$$\|\vec{u}\| = \sqrt{\vec{u} \cdot \vec{u}} = \sqrt{245} \quad \checkmark = 7\sqrt{5}$$

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{\pi^2 + 10} \quad \checkmark$$

$$\vec{u} \cdot \vec{v} = (15 \cdot \pi) + (-2 \cdot 3) + (4 \cdot -1) = 15\pi - 6 - 4 = 15\pi - 10 \quad \checkmark$$

1.2.9

$$\vec{u} = -\vec{i} + 3\vec{j} + \vec{k} = (-1, 3, 1)$$

$$\vec{v} = -2\vec{i} - 3\vec{j} - 7\vec{k} = (-2, -3, -7)$$

$$\|\vec{u}\| = \sqrt{\vec{u} \cdot \vec{u}} = \sqrt{(-1)^2 + (3)^2 + (1)^2} = \sqrt{11} \quad \checkmark$$

$$\|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}} = \sqrt{(-2)^2 + (-3)^2 + (-7)^2} = \sqrt{4 + 9 + 49} = \sqrt{62} \quad \checkmark$$

$$\vec{u} \cdot \vec{v} = (-1)(-2) + (3)(-3) + (1)(-7) = 2 - 9 - 7 = -14 \quad \checkmark$$

Homework #1

~~4/7~~

5/8

1.2.12

$$\vec{v} = (2, 3), \vec{w} = (w_1, w_2)$$

$$\vec{v} \cdot \vec{w} = 0 \text{ by perpendicularity}$$

$$\Rightarrow 2 \cdot w_1 + 3 \cdot w_2 = 0 \Rightarrow w_2 = -\frac{2}{3}w_1$$

$$\text{so } \vec{w} = (w_1, -\frac{2}{3}w_1)$$

$$\|\vec{w}\| = 5, \text{ so } 5 = \|\vec{w}\| = \sqrt{(w_1)^2 + (-\frac{2}{3}w_1)^2} = \sqrt{\frac{13}{9}w_1^2}$$

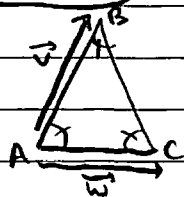
$$\text{so } \frac{13}{9}w_1^2 = 25 \Rightarrow w_1^2 = \frac{225}{13}, w_1 = \pm \frac{15}{\sqrt{13}}$$

$$\Rightarrow \vec{w} = \left(\frac{15}{\sqrt{13}}, -\frac{10}{\sqrt{13}} \right) \text{ or } \vec{w} = \left(-\frac{15}{\sqrt{13}}, \frac{10}{\sqrt{13}} \right)$$

1.2.15

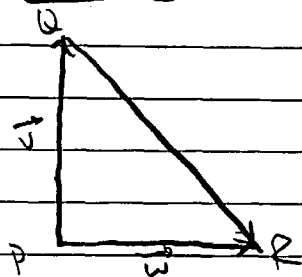
We know that $\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cos \theta$, so if $\vec{v} \cdot \vec{w} = -\|\vec{v}\| \cdot \|\vec{w}\|$ then $\cos \theta = -1$ and $\theta = 180^\circ$, such that \vec{v} and \vec{w} are parallel and pointing in opposite directions

1.2.23



$$\begin{aligned} \vec{v} \cdot \vec{w} &= \|\vec{v}\| \cdot \|\vec{w}\| \cos \theta \\ &= 1 \cdot 1 \cdot \cos(60) \\ &= \frac{1}{2} \checkmark \end{aligned}$$

1.2.27



Let PQR be a triangle with sides \vec{PQ} and \vec{PR} given by \vec{v} and \vec{w} , and assume that $\|\vec{PQ}\|^2 + \|\vec{PR}\|^2 = \|\vec{QR}\|^2$. Since we know $\|\vec{QR}\| = \|\vec{w} - \vec{v}\|$, we can rewrite this as:

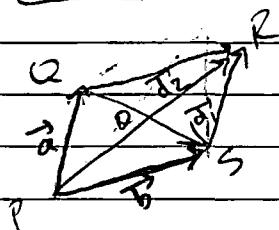
$$\|\vec{v}\|^2 + \|\vec{w}\|^2 = \|\vec{w} - \vec{v}\|^2$$

Since $\|\vec{a}\|^2 = \vec{a} \cdot \vec{a}$ for all vectors \vec{a} , we can further rewrite this as:

$$\begin{aligned} \vec{v} \cdot \vec{v} + \vec{w} \cdot \vec{w} &= (\vec{w} - \vec{v}) \cdot (\vec{w} - \vec{v}) \\ &= \vec{w} \cdot (\vec{w} - \vec{v}) - \vec{v} \cdot (\vec{w} - \vec{v}) = \vec{w} \cdot \vec{w} - 2\vec{w} \cdot \vec{v} + \vec{v} \cdot \vec{v} \text{ by the distributive property of the inner product. Then,} \end{aligned}$$

$$\begin{aligned} \vec{v} \cdot \vec{v} + \vec{w} \cdot \vec{w} &= \vec{w} \cdot \vec{w} - 2\vec{w} \cdot \vec{v} + \vec{v} \cdot \vec{v} \\ \Rightarrow 0 &= -2\vec{w} \cdot \vec{v} \Rightarrow \vec{w} \cdot \vec{v} = 0 \text{ which implies } \vec{v} \text{ and } \vec{w} \text{ are orthogonal; they were the legs of the triangle } PQR, \text{ so } PQR \text{ must be a right triangle. } \checkmark \end{aligned}$$

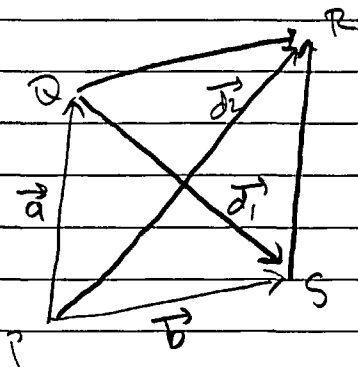
#4



Let PQRS be a parallelogram with sides given by \vec{a} and \vec{b} , such that its diagonals \vec{d}_1 and \vec{d}_2 are given by $\vec{d}_1 = \vec{b} - \vec{a}$ and $\vec{d}_2 = \vec{b} + \vec{a}$.

① \Rightarrow : Assume $\|\vec{b}\| = \|\vec{a}\|$, i.e. PQRS is a rhombus. Then, $\vec{d}_1 \cdot \vec{d}_2 = (\vec{b} - \vec{a}) \cdot (\vec{b} + \vec{a}) = \vec{b} \cdot (\vec{b} + \vec{a}) - \vec{a} \cdot (\vec{b} + \vec{a})$
 $= \vec{b} \cdot \vec{b} - \vec{a} \cdot \vec{a}$, by the distributive property of inner products
 $= \|\vec{b}\|^2 - \|\vec{a}\|^2 = \|\vec{b}\|^2 - \|\vec{b}\|^2 = 0$, so the diagonals \vec{d}_1 and \vec{d}_2 must be perpendicular.

② \Leftarrow : Assume the diagonals are perpendicular. Then, $\vec{d}_1 \cdot \vec{d}_2 = 0$. But $\vec{d}_1 \cdot \vec{d}_2 = (\vec{b} - \vec{a}) \cdot (\vec{b} + \vec{a}) = \vec{b} \cdot \vec{b} - \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{a} = \vec{b} \cdot \vec{b} - \vec{a} \cdot \vec{a}$. As a result, $\vec{b} \cdot \vec{b} - \vec{a} \cdot \vec{a} = 0$ and $\vec{b} \cdot \vec{b} = \vec{a} \cdot \vec{a}$, which implies $\|\vec{a}\|^2 = \|\vec{b}\|^2$ and $\|\vec{a}\| = \|\vec{b}\|$, so the sides of PQRS must all be equal length and thus PQRS is a rhombus.



Larger
Picture

#5

$$\vec{a} = (a_1, a_2, a_3) \quad \vec{b} = (b_1, b_2, b_3) \quad \vec{c} = (c_1, c_2, c_3)$$

$$\begin{aligned} 1) \quad \vec{a} \cdot (\vec{b} + \vec{c}) &= \vec{a} \cdot (b_1 + c_1, b_2 + c_2, b_3 + c_3) = a_1(b_1 + c_1) + a_2(b_2 + c_2) + a_3(b_3 + c_3) \\ &= a_1b_1 + a_1c_1 + a_2b_2 + a_2c_2 + a_3b_3 + a_3c_3 \\ &= (a_1b_1 + a_2b_2 + a_3b_3) + (a_1c_1 + a_2c_2 + a_3c_3) \\ &= \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} \end{aligned}$$

$$\begin{aligned} 2) \quad (\vec{a} + \vec{b}) \cdot \vec{c} &= (a_1 + b_1, a_2 + b_2, a_3 + b_3) \cdot (c_1, c_2, c_3) \\ &= (a_1 + b_1)c_1 + (a_2 + b_2)c_2 + (a_3 + b_3)c_3 \\ &= a_1c_1 + b_1c_1 + a_2c_2 + b_2c_2 + a_3c_3 + b_3c_3 \\ &= (a_1c_1 + a_2c_2 + a_3c_3) + (b_1c_1 + b_2c_2 + b_3c_3) \\ &= \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c} \end{aligned}$$