(1) $F(x) \approx L(x) = F'(a)(x-a) + F(a)$

$= 7(x-3) + 2$

$= 7(2.99-3) + 2$

$= -.07 + 2$

$= 1.93$

(2) $g(x) = x^{3/4} - 2x^{1/4}$ on $[0, 4]$

$g'(x) = \frac{3}{4}x^{-1/4} - 2(\frac{1}{4})x^{-3/4}$

$g'(x) = \frac{3}{4}x^{-1/4} - \frac{1}{2}x^{-3/4}$

• $g'(x)$ does not exist at 0

• $g'(x) = 0$ when

$0 = \frac{3}{4}x^{-1/4} - \frac{1}{2}x^{-3/4}$

$\frac{1}{2}x^{-3/4} = \frac{3}{4}x^{-1/4}$

$x^{-1/2} = \frac{3}{2}$

$x^{1/2} = \frac{2}{3}$

$x = \frac{4}{9}$

Critical Numbers: 0, 4/9

$g(0) = 0$

$g(4/9) = (\frac{4}{9})^{3/4} - 2(\frac{4}{9})^{1/4}$

$= \frac{2\sqrt{2}}{3\sqrt{3}} - \frac{2\sqrt{2}}{\sqrt{3}}$

$= \frac{-4\sqrt{2}}{3\sqrt{3}}$

Endpoints: 0, 4

$g(0) = 0$

$g(4) = (4)^{3/4} - 2(4)^{1/4}$

$= 2\sqrt{2} - 2\sqrt{2}$

$= 0$

Maximum: 0

Minimum: $\frac{-4\sqrt{2}}{3\sqrt{3}}$

(3) $f(x) = x^{1/3}$ is not differentiable at 0, hence not on $(-1, 1)$

However, $f(x)$ is continuous on $[0, 2]$ and differentiable on $(0, 2)$ $\rightsquigarrow$ $f(x)$ satisfies the hypotheses of MVT on $[0, 2]$.

Want c such that

$$f'(c) = \frac{f(2) - f(0)}{2 - 0} = \frac{2^{1/3} - 0}{2} = \frac{2^{1/3}}{2} = \frac{1}{2^{2/3}}$$

Now $f'(x) = \frac{1}{3} \cdot \frac{1}{x^{2/3}}$, So we want $\frac{1}{2^{2/3}} = \frac{1}{3} \cdot \frac{1}{x^{2/3}}$

$$3x^{2/3} = 2^{2/3}$$

$$x^{2/3} = \frac{2^{2/3}}{3}$$

$$x = \frac{2}{3^{3/2}}$$

$$x = \frac{2}{3\sqrt{3}}$$

$$\boxed{c = \frac{2}{3\sqrt{3}}}$$

④ $f(x) = \dfrac{\sqrt{1+x^2}}{x}$

<u>Intercepts</u> <u>y-intercept</u> ⇝ $f(0)$ does not exist

None

<u>x-intercept</u> ⇝ $f(x) \neq 0$ for any x

None

<u>Asymptotes</u> <u>Vertical</u> $x=0$

$\lim_{x \to 0^-} f(x) = -\infty \qquad \lim_{x \to 0^+} f(x) = +\infty$

<u>Horizontal</u>

$y=1 \qquad \lim_{x \to \infty} \dfrac{\sqrt{1+x^2}}{x} = \lim_{x \to \infty} \sqrt{\dfrac{1}{x^2}+1} = 1$

$y=-1 \qquad \lim_{x \to -\infty} \dfrac{\sqrt{1+x^2}}{x} = \lim_{x \to -\infty} -\sqrt{\dfrac{1}{x^2}+1} = -1$

<u>Symmetry</u> $f(-x) = \dfrac{\sqrt{1+(-x)^2}}{-x} = -\dfrac{\sqrt{1+x^2}}{x} = -f(x)$ <u>f is odd</u>

<u>Increasing/ Decreasing</u> $f'(x) = \dfrac{\frac{x}{\sqrt{1+x^2}} \cdot x - \sqrt{1+x^2}}{x^2} = \dfrac{x^2-(1+x^2)}{x^2\sqrt{1+x^2}} = \dfrac{-1}{x^2\sqrt{1+x^2}}$ } Negative everywhere

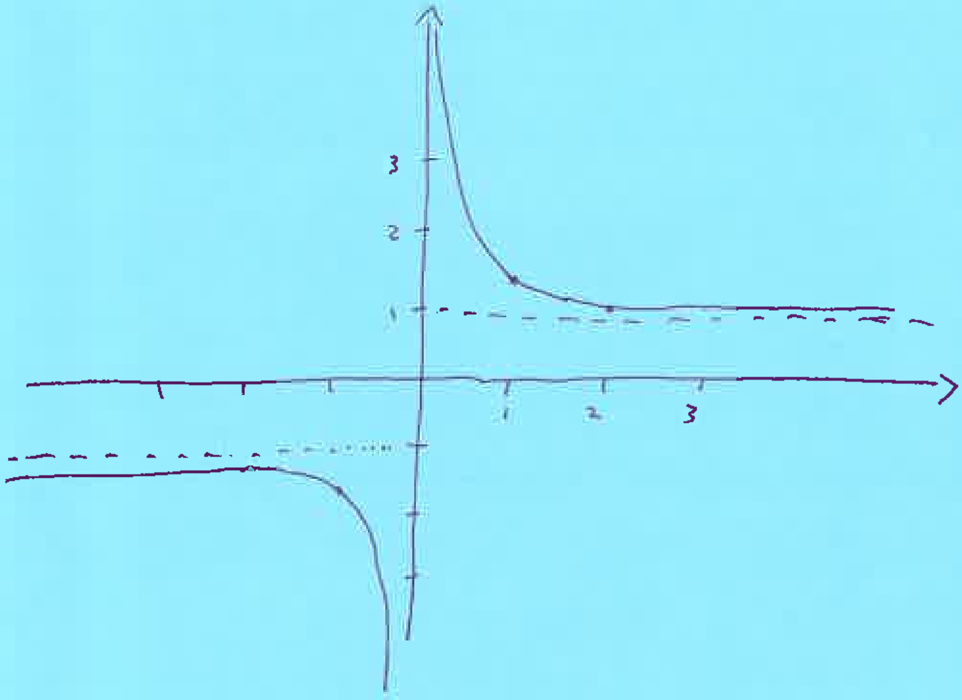
$f'(x)$ is decreasing everywhere ⇝ no local maxima or minima

<u>Concavity</u> $f'(x) = \dfrac{-1}{\sqrt{x^4+x^6}} = -(x^4+x^6)^{-1/2}$

$f''(x) > 0$ when x positive and $f''(x) < 0$ when x negative.

$f''(x) = \frac{1}{2}(x^4+x^6)^{-3/2} \cdot (4x^3+6x^5)$

$\rightsquigarrow f(x)$ concave up on $(-\infty, 0)$ and concave down on $(0, \infty)$



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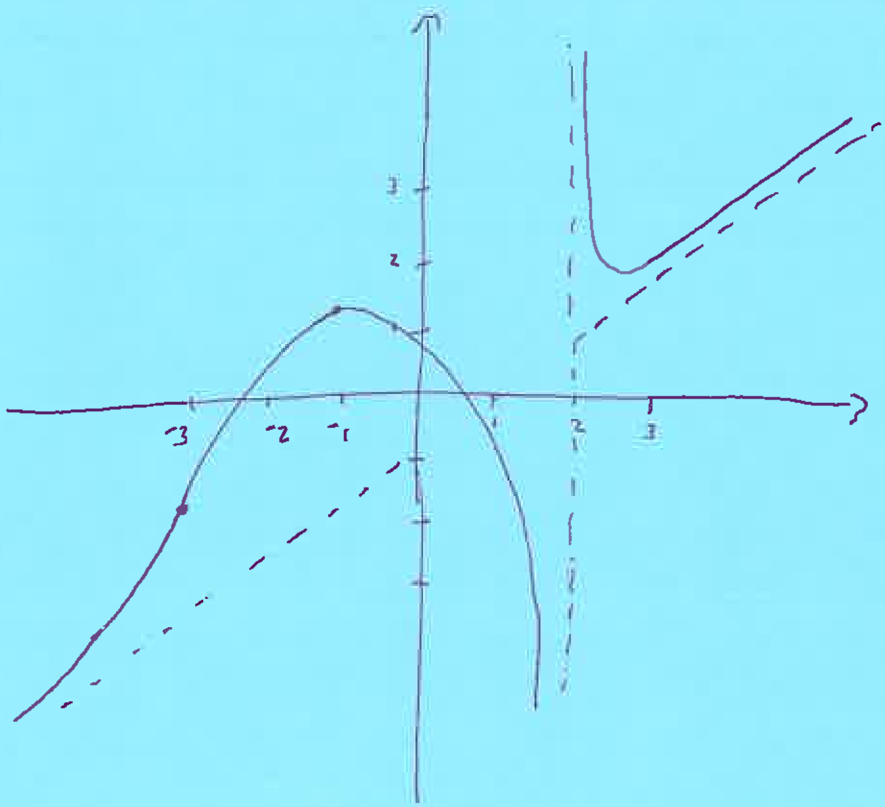
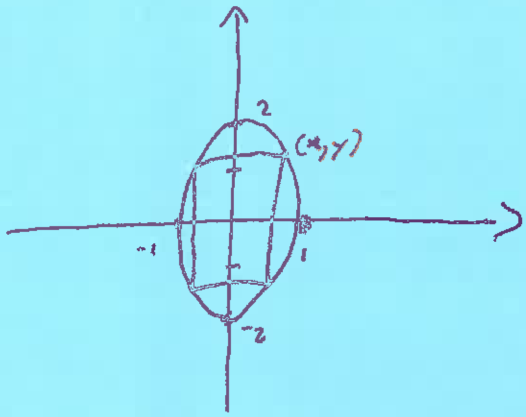


⑥



Area $= 2x(2y) = 4xy$

$A^2 = \theta = 16x^2y^2$

<u>Constraint</u> $4x^2+y^2=4$

$y^2 = 4-4x^2$

$\theta(x) = 16x^2(4-4x^2)$

$\theta(x) = 64(x^2-x^4)$

$\theta'(x) = 64(2x-4x^3)$

$0 = 64(2x-4x^3)$

$0 = x(2-4x^2)$

Range of possible x-values $[0,1]$

$x=0 \quad 2=4x^2$

$\frac{1}{2} = x^2$

$\frac{1}{\sqrt{2}} = x$

$x=0 \rightsquigarrow$ Trivial $\quad x=1 \rightsquigarrow$ Trivial

$x = \frac{1}{\sqrt{2}} \rightsquigarrow \quad y^2 = 4-4\left(\frac{1}{\sqrt{2}}\right)^2$

$y^2 = 4-2$

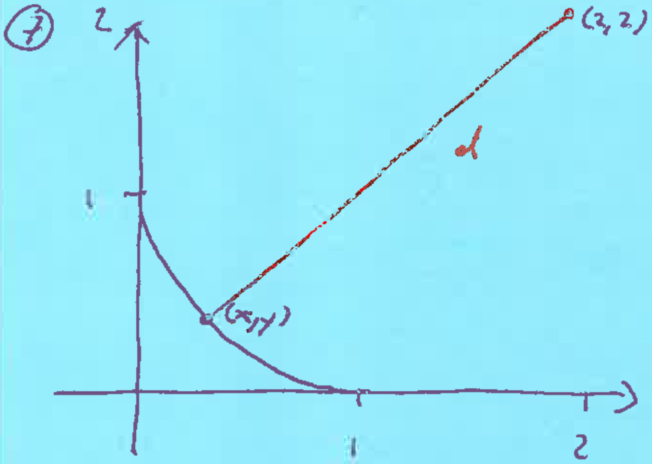
$y^2 = 2$

$y = \sqrt{2}$

$A = 4xy = 4\left(\frac{1}{\sqrt{2}}\right)(\sqrt{2}) = 4$

Rectangle has dimensions $\frac{2}{\sqrt{2}}$ by $2\sqrt{2}$, or $\sqrt{2}$ by $2\sqrt{2}$.

(7)



$$d = \sqrt{(x-2)^2 + (y-2)^2}$$

$$D = d^2 = (x-2)^2 + (y-2)^2$$

<u>Constraint</u> $\sqrt{x} + \sqrt{y} = 1$

$$\sqrt{y} = 1 - \sqrt{x}$$

$$y = (1-\sqrt{x})^2$$

$$D(x) = (x-2)^2 + ((1-\sqrt{x})^2 - 2)^2 \quad \text{on} \quad 0 \le x \le 1.$$

(8) $F(x) = \dfrac{x+1}{3-x}$

$$L_3 = \Delta x\,(F(-1) + F(0) + F(1))$$

$$= 1\,(0 + \tfrac{1}{3} + 1)$$

$$= \frac{4}{3}$$

Underestimate

(9) $F(x) = \int_{x^3}^{3} \sqrt{t^2+1}\, dt$

$$= -\int_{3}^{x^3} \sqrt{t^2+1}\, dt$$

$$F'(x) = -\sqrt{(x^3)^2+1} \cdot 3x^2$$

$$= -3x^2\sqrt{x^6+1}$$

(10) $\int_4^5 \left(\frac{3+x}{\sqrt{x}} + \frac{\pi}{4}\sec\left(\frac{\pi}{4}x\right)\tan\left(\frac{\pi}{4}x\right) \right) dx$

$$= \int_4^5 \left(3x^{-1/2} + x^{1/2}\right) dx + \int_4^5 \frac{\pi}{4}\sec\left(\frac{\pi}{4}x\right)\tan\left(\frac{\pi}{4}x\right) dx$$

$$= \left[6x^{1/2} + \frac{2}{3}x^{3/2}\right]_4^5 + \sec\left(\frac{\pi}{4}x\right)\Big|_4^5$$

$$= \left[\left(6\sqrt{5} + \frac{2}{3}(5\sqrt{5})\right) - \left(12 + \frac{2}{3}(8)\right)\right] + \left[\sec\pi - \sec\frac{5\pi}{4}\right]$$

$$= \frac{28\sqrt{5}}{3} - \frac{52}{3} - 1 + \sqrt{2}$$

$$= \frac{28\sqrt{5}}{3} - \frac{55}{3} + \sqrt{2}$$

$$= \frac{28\sqrt{5} + 3\sqrt{2} - 55}{3}$$

(11) $\int_2^7 3F(x)dx = 3\int_2^7 F(x)dx$

$= 3\left[\int_1^7 F(x)dx - \int_1^2 F(x)dx\right]$

$= 3\left[\int_1^7 F(x)dx - \left[\int_1^3 F(x)dx - \int_2^3 F(x)dx\right]\right]$

$= 3\left[3 - \left[-1 - 7\right]\right]$

$= 3\left[3 + 8\right]$

$= 33$

(12) $h(x) = \sin x + 3x(1 - x^{1/3}) = \sin x + 3x - 3x^{4/3}$

$H(x) = -\cos x + \frac{3}{2}x^2 - \frac{9}{7}x^{7/3} + C$] Most general antiderivative

Contains $(0, 4)$

$4 = H(0) = -\cos(0) + \frac{3}{2}(0)^2 - \frac{9}{7}(0)^{7/3} + C$

$4 = -1 + 0 + 0 + C$

$3 = C$

$H(x) = -\cos x + \frac{3}{2}x^2 - \frac{9}{7}x^{7/3} + 3$

(13) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \cos\left(3 + \frac{2i}{n}\right)$

$\Delta x = \frac{2}{n} = \frac{b-a}{n}$ so $b - a = 2$

$x_i = 3 + \frac{2i}{n}$ so $a = 3 \leadsto b = 5$

$$\int_3^5 \cos x \, dx$$