1. Suppose you know that $f(3) = 2$ and $f'(3) = 7$. Approximately what is $f(2.99)$?

2. Find the absolute maximum and minimum of $g(x) = x^{\frac{3}{4}} - 2x^{\frac{1}{4}}$ over the interval $[0, 4]$.

3. On which of the two intervals $[-1, 1]$ and $[0, 2]$ does $f(x) = x^{\frac{1}{3}}$ satisfy the hypotheses of the Mean Value Theorem? For the interval you picked, find the value of c at which the conclusion of the Mean Value Theorem is true.

4. Consider the function $f(x) = \frac{\sqrt{1+x^2}}{x}$. Find its intercepts, asymptotes, symmetry, intervals of increase and decrease, local maxima and minima, and intervals of concavity. Then sketch a graph.

5. Draw a function $f(x)$ such that

   - $f(x)$ has a vertical asymptote at $x = 2$ and a slant asymptote at $y = x - 1$.
   - $f'(x)$ is positive on $(-\infty, -1)$ and $(3, \infty)$, and negative on $(-1, 2)$ and $(2, 3)$.
   - $f(x)$ is positive on $(-\infty, -3)$ and $(2, \infty)$, and negative on $(-3, 2)$.

6. What are the dimensions of the largest rectangle that can be inscribed in the ellipse $4x^2 + y^2 = 4$? [Hint: You can optimize the square of the area function.]

7. Give a single-variable function you could optimize to find the point on the curve $\sqrt{x} + \sqrt{y} = 1$ closest to $(2, 2)$. (Do not actually do the optimization.) What is the range of values of the independent variable you should consider?

8. Estimate the area under the curve $f(x) = \frac{x+1}{3-x}$ over the interval $[-1, 2]$ using the left-hand endpoints of three rectangles of equal width. Is your answer an overestimate, underestimate, or not obviously either?

9. What is the derivative of the function $F(x) = \int_{x^3}^{3} \sqrt{t^2 + 1}dt$?

10. Find the integral $\int_4^5 \left(\frac{3+x}{\sqrt{x}} + \frac{\pi}{4} \sec(\frac{\pi}{4}x) \tan(\frac{\pi}{4}x) \right) dx$.

11. Suppose you know that $\int_2^3 f(x)dx = 7$, $\int_1^7 f(x)dx = 3$, and $\int_1^3 f(x)dx = -1$. What is $\int_2^7 3f(x)dx$?

12. What is the most general antiderivative of $h(x) = \sin x + 3x(1 - x^{\frac{1}{3}})$? What is the particular antiderivative of the curve that contains the point $(0, 4)$?

13. Find a definite integral which can be evaluated using the limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \cos\left(3 + \frac{2i}{n}\right)$$

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