1a) $$\lim_{x\to 0} \frac{\sqrt{x^2+1} - \sqrt{1-x^2}}{x^2}\left(\frac{\sqrt{x^2+1} + \sqrt{1-x^2}}{\sqrt{x^2+1} + \sqrt{1-x^2}}\right)$$

$$= \lim_{x\to 0} \frac{(x^2+1) - (1-x^2)}{x^2(\sqrt{x^2+1} + \sqrt{1-x^2})}$$

$$= \lim_{x\to 0} \frac{2\cancel{x^2}}{\cancel{x^2}(\sqrt{x^2+1} + \sqrt{1-x^2})}$$

$$= \frac{2}{2}$$

$$= 1$$

(1b) $$\lim_{x \to 2^-} \frac{|x^2-3x+2| \cos(2x)}{x^2-3x+2} = \lim_{x \to 2^-} \frac{-(x^2-3x+2)\cos 2x}{x^2-3x+2}$$

$$= \lim_{x \to 2^-} -\cos(2x)$$

$$= -\cos(4)$$

<u>Note</u> $x^2-3x+2 = (x-1)(x-2)$

$$|x^2-3x+2| = \begin{cases} x^2-3x+2 & x \le 1 \\ -(x^2-3x+2) & 1 \le x \le 2 \\ x^2-3x+2 & x > 2 \end{cases}$$

(1c) $$\lim_{x \to 0} \frac{\sin(x^2-x)}{\tan(3x)} = \lim_{x \to 0} \sin(x^2-x) \cdot \frac{\cos(3x)}{\sin(3x)}$$

$$= \lim_{x \to 0} \frac{\sin(x^2-x)}{x^2-x} \cdot \frac{3x}{\sin 3x} \cdot \cos(3x) \cdot \frac{x^2-x}{3x}$$

$$= (1)\cdot(1)\cdot(1)\cdot\frac{-1}{3}$$

$$= \frac{-1}{3}$$

(1d) Notice that $-1 \le \cos\left(\frac{1}{x}\right) \le 1$, so $-x \le x\cos\left(\frac{1}{x}\right) \le x$. Now $\lim_{x \to 0} x = 0 = \lim_{x \to 0} -x$, so by the Squeeze Thm, $\lim_{x \to 0} x\cos\left(\frac{1}{x}\right) = 0$.

(2a) $F(x) = x^2 \sin(x^2)$

$$F'(x) = 2x\sin(x^2) + x^2(2x)\cos(x^2)$$

$$= 2x\sin(x^2) + 2x^3\cos(x^2)$$

(2b) $g(x) = \cos(\sec(x-1))$

$$g'(x) = -\sin(\sec(x-1)) \cdot [\sec(x-1)\tan(x-1)]$$

(2c) $h(x) = \dfrac{x^2+2}{\sqrt{x-x^3}}$

$$h'(x) = \frac{2x\sqrt{x-x^3} - \frac{1-3x^2}{2\sqrt{x-x^3}}(x^2+2)}{x-x^3}$$

(3) $F(x) = \begin{cases} \dfrac{x^2-4}{x-2} & x < 2 \\ ax^2+bx+3 & 2 \le x < 3 \\ 2x-a-b & x \ge 3 \end{cases}$

Need $\lim\limits_{x\to 2^-} F(x) = \lim\limits_{x \to 2^+} F(x)$ and $\lim\limits_{x\to 3^-} F(x) = \lim\limits_{x\to 3^+} F(x)$

$$\lim_{x\to 2^-} \frac{x^2-4}{x-2} = \lim_{x\to 2^+} ax^2+bx+3$$

$$\lim_{x\to 2^-} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = \lim_{x\to 2^+} ax^2+bx+3$$

$$4 = a(2)^2 + b(2) + 3$$

$$0 = a2^2 + b(2) - 1$$

$$0 = 4a + 2b - 1$$

$$\lim_{x\to 3^-} ax^2+bx+3 = \lim_{x\to 3^+} 2x-a-b$$

$$9a+3b+3 = 6-a-b$$

$$0 = -10a - 4b + 3$$

$$+\ (0 = 8a + 4b - 2)$$

$$0 = -2a + 1$$

$$2a = 1$$

$$\boxed{a = \tfrac{1}{2}}$$

$$0 = 4(\tfrac{1}{2}) + 2b - 1$$

$$-1 = 2b$$

$$\boxed{\tfrac{-1}{2} = b}$$

④ Want $|F(x) - 2| < 1$

$\iff |\sqrt{x-1} - 2| < 1$

$\iff -1 < \sqrt{x-1} - 2 < 1$

$\iff 1 < \sqrt{x-1} < 3$

$\iff 1 < x - 1 < 9$

$-3 < x - 5 < 5$

$\uparrow$

Implied by $|x-5| < 3$. So we can take $\delta = 3$.

⑤ $\lim\limits_{h \to 0} \dfrac{F(x+h) - F(x)}{h} = \lim\limits_{h \to 0} \dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

$= \lim\limits_{h \to 0} \dfrac{x^2 - (x+h)^2}{h(x+h)^2(x)^2}$

$= \lim\limits_{h \to 0} \dfrac{x^2 - (x^2 + 2xh + h^2)}{h(x+h)^2 x^2}$

$= \lim\limits_{h \to 0} \dfrac{-(2x+h)}{(x+h)^2 x^2}$

$= \dfrac{-2x}{x^4}$

$= \dfrac{-2}{x^3}$

⑥ $x\cos x = x^2 - 1$ in $[0, \pi]$

$$F(x) = x\cos x + 1 - x^2$$

$$F(0) = 1 \qquad F(\pi) = \pi(\cos\pi) + 1 - \pi^2 = \pi + 1 - \pi^2 < 0$$

Since f is continuous on $[0,\pi]$, we can apply the intermediate value theorem: since $f(0) > 0$ and $f(\pi) < 0$, we see that there exists $c \in (0,\pi)$ such that $f(c) = 0$. Then c solves the original equation.

⑦ $y\sin(2x) = x\cos(2y)$ at $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$

$$\frac{dy}{dx}\sin(2x) + y(\cos(2x))(2) = \cos(2y) + x(-\sin(2y))(2)\left(\frac{dy}{dx}\right)$$

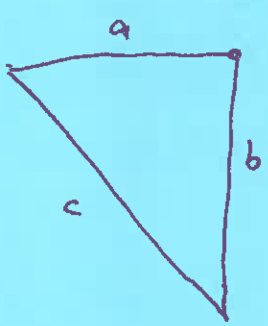
$$0 + \frac{\pi}{4}(1)(2) = 0 + \frac{\pi}{2}(-1)(2)\frac{dy}{dx}$$

$$\frac{\pi}{2} = -\pi\frac{dy}{dx}$$

$$-\frac{1}{2} = \frac{dy}{dx}$$

$$y = \frac{-1}{2}x + \frac{\pi}{2}$$

⑧



a - distance westward bound car has travelled

b - distance southward bound car has traveled

c - distance between the cars

After two hours: $\qquad a^2 + b^2 = c^2$

$a = 50$

$b = 120$

$c = 130$ ← Pythagorean triple

$$\cancel{2}a\left(\frac{da}{dt}\right) + \cancel{2}b\left(\frac{db}{dt}\right) = \cancel{2}c\left(\frac{dc}{dt}\right)$$

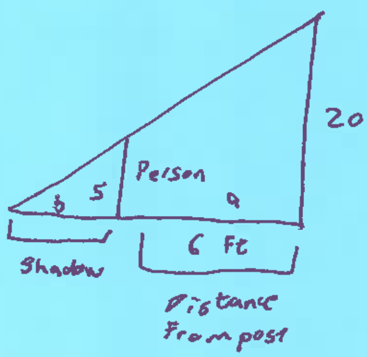
$$50(25) + 120(60) = 130\,\frac{da}{dt}$$

$$2(25)^2 + 2(60)^2 = 2(65)\,\frac{dc}{dt}$$

$$65 = \frac{dc}{dt}$$

(25 − 60 − 65) is a (5−12−13) triple

(9)



$$\frac{da}{dt} = -4\ \text{ft}/\text{s}$$

$$\frac{20}{a+b} = \frac{5}{b}$$

$$5(a+b) = 20b$$

$$5a = 15b$$

$$a = 3b$$

$$\frac{da}{dt} = 3\,\frac{db}{dt}$$

$$-4\,\frac{\text{ft}}{\text{s}} = 3\,\frac{db}{db}$$

$$\frac{-4}{3}\,\frac{\text{ft}}{\text{s}} = \frac{db}{dt}$$

(10) $s(t) = 100 + 20t - 26t^2$

(a) $$t = \frac{-20 \pm \sqrt{400 - 4(100)(-26)}}{-52}$$

$$= \frac{5}{13} + \frac{\sqrt{400(27)}}{52}$$

$$= \frac{5}{13} + \frac{5\sqrt{27}}{13}$$

$$= \frac{5(1+\sqrt{27})}{13} \approx 2.38\ \text{s}$$

(b) $v(t) = 20 - 52t$

$-58 = 20 - 52t$

$-78 = -52t$

$1.5 = t$

$s(1.5) = 100 + 30 + -26\left(\frac{3}{2}\right)^2$

$= 130 + \frac{117}{2}$

$= 188.5$

$\boxed{s(1.5) = 188.5}$

(c) $0 = v(t)$

$0 = 20 - 52t$

$52t = 20$

$t = \frac{5}{13}$

$s\left(\frac{5}{13}\right) = 100 + 20\left(\frac{5}{13}\right) - 26\left(\frac{5}{13}\right)^2$

$= 100 + \frac{100}{13} - \frac{650}{189}$

$= 100 + \frac{100}{13} - \frac{50}{13}$

$= \boxed{\frac{1350}{13} \text{ ft}}$